Fundamentals in Nuclear Physics 原子核基礎

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Nuclear decays and fundamental interactions

Four fundamental interactions

interaction 相互作用	exchanged particle (gauge boson)	decay 壞変
gravity 重力	graviton 重力子	
weak 弱い相互作用	W±, Z ⁰	beta decay
electromagnetic 電磁相互作用	photon 光子	gamma decay
strong 強い相互作用	gluon グルーオン	
nuclear force 核力	pion and other hadrons	



壞変(崩壞)速度 自然幅 Decay rate, natural width

probability to decay in an interval dt

$$dP = \frac{dt}{\tau} = \lambda dt$$
 decay rate 壞変(崩壞)速度 mean life time 平均寿命

number of unstable nuclei $N(t) = N(t=0)e^{-t/\tau}$ half life 半減期 $t_{1/2} = (\ln 2)\tau = 0.693\tau$

⁷Li (7.459 MeV) \rightarrow n⁶Li, ³H⁴He $\tau = 6 \times 10^{-21}$ sec ⁷⁶Ge \rightarrow ⁷⁶Se 2e⁻ $2\bar{\nu}_e$ $t_{1/2} = 1.78 \times 10^{21}$ yr $> 10^{11} \times$ (age of universe) !

An unstable particle has an energy uncertainty or "natural width"

$$\Gamma = \hbar \lambda = \frac{\hbar}{\tau} = \frac{6.58 \times 10^{-22} \,\mathrm{MeV \ sec}}{\tau}$$

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分岐比 Branching ratio

• Often, an unstable state (nucleus, isotope) has more than one decay channels.



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_{壊変図} Decay diagram



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Measurement of half life



$\tau > 10^8$ yr (α decay, double β decay)



Basdevant, Rich, and Spiro, "Fundamentals in Nuclear Physics" (Springer, 2005)

- still present on Earth
- can be chemically and isotopically isolated in macroscopic quantity
- detected decays, quantity → lifetime

$$^{00}Mo \rightarrow {}^{100}Ru \, 2e^{-} \, 2\bar{\nu}_{e}$$
 double β decay half-life: (0.95±0.11)×10¹⁹ yr

10 min < τ < 10⁸ yr (α decay, β decay)

- no longer present on Earth and must be produced in nuclear reactions
- purify chemically or isotopically
- detect decays and derive τ

10^{-10} s < τ < 10^3 s (α decay, β decay, γ decay)

- chemical and isotopic purification impossible
- particles produced in nuclear reactions, slowed down, and stopped
- detect decays and derive τ
- $\tau < 10^{-10}$ s (γ decay, dissociation)
- standard timing techniques not applicable
- a variety of ingenious techniques: Doppler-shift attenuation method, Mössbauer spectroscopy

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Formula for decay rates



decay rate

probability per unit time that *a* decays into *f* 粒子 *a* が単位時間に状態 *f* に壊変する確率

$$\lambda_{a \to f} = \frac{2\pi}{\hbar} \frac{|\langle f|T|a \rangle|^2}{|\langle f|T|a \rangle|^2} \delta \left(Mc^2 - \sum_j E_j \right)$$
Fermi's golden rule
clement
選移行列要素 energy conservation
 $x \neq \mu \neq - Re_f$



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Emitted gamma rays are not resonantly re-absorbed by other nuclei in gases

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電気双極子遷移 Electric-dipole transitions

Classical image 古典電磁気学的なイメージ radiation from an oscillating electric dipole 振動する電気双極子からの古典的な放射 Quantum mechanically 量子力学的には rate $\lambda_{i \to f} = \frac{4\alpha}{3} \frac{q^2}{e^2} \frac{E_{\gamma}^3}{\hbar^3 c^2} |\langle f | \mathbf{r} | i \rangle|^2$ fine-structure constant $\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \simeq \frac{1}{137}$ $\langle f | \mathbf{r} | i \rangle = \int d^3 \mathbf{r} \psi_f^*(\mathbf{r}) \mathbf{r} \psi_i(\mathbf{r})$ Atomic transition

$$\frac{\hbar\omega \sim eV}{\hbar\omega \sim eV} \quad \langle r \rangle \sim 10^{-10} \,\mathrm{m} \quad \tau \sim 10^{-9} - 10^{-7} \,\mathrm{s} \quad \Gamma = \hbar/\tau \sim 10^{-7} \,\mathrm{eV} \ll \hbar\omega \\ \gg E_R = E_{\gamma}^2/(2m_A c^2) \sim 10^{-9} \,\mathrm{eV}$$
Nuclear transition
$$\langle r \rangle \sim A^{1/3} 10^{-15} \,\mathrm{m} \quad \Longrightarrow \quad \lambda(E1) \sim \frac{\alpha E_{\gamma}^3}{\hbar} \left(\frac{A^{1/3} \,\mathrm{fm}}{\hbar c}\right)^2$$

 $E_{\gamma} \sim \text{MeV} \quad \tau \sim 10^{-17} - 10^{-15} \text{ s} \quad \Gamma \sim 10 \text{ eV} \ll E_{\gamma}$

^{多重極遷移} Higher multi-pole transitions

Often, electric-dipole (E1) decay is forbidden. $\langle f | \mathbf{r} | i \rangle = 0$

may still decay radiatively by higher-order and slower processes

Table 4.1. Selection rules for radiative transitions				
type	symbol	angular momentum change $ \Delta J \leq$	parity change	
alastria dinala	F 1	1		
electric dipole		1	yes	
magnetic dipole	M1	1	no	
electric quadrupole	E2	2	no	
magnetic quadrupole	M2	2	yes	
electric octopole	E3	3	yes	
magnetic octopole	M3	3	no	
electric 16-pole	E4	4	no	
magnetic 16-pole	M4	4	yes	

Basdevant, Rich, and Spiro, "Fundamentals in Nuclear Physics" (Springer, 2005)



Lifetime of excited nuclear states as a function of E_{γ} for various multipoles

Basdevant, Rich, and Spiro, "Fundamentals in Nuclear Physics" (Springer, 2005)

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Internal conversion

An excited nucleus can interact with an electron in one of the lower atomic orbitals, causing the electron to be emitted (ejected) from the atom.

s-electrons have finite probability density at the nuclear position.



The electron may couple to the excited state of the nucleus and take the energy of the nuclear transition <u>directly</u>, without an <u>intermediate gamma ray</u>.



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電子運動量 electron momentum

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メスバウアー効果 Mössbauer effect

recoil energy (energy loss) 反跳エネルギー(エネルギー損失)

$$E_R = \frac{E_\gamma^2}{2m_A c^2}$$

Emitted gamma rays are not resonantly re-absorbed by other nuclei in gases.



Inverse transition (resonant re-absorption) possible when

- nuclear recoil is suppressed in a crystal ("very very large m_A") ← Mössbauer effect (discovered in 1957)
- the excited nucleus decays in flight with the Doppler effect compensating the nuclear recoil

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メスバウアー分光による寿命測定 **Mössbauer spectroscopy**



Basdevant, Rich, and Spiro, "Fundamentals in Nuclear Physics" (Springer, 2005)

Doppler-shift attenuation method





http://ishiken.free.fr/english/lecture.html Fundamentals in Nuclear Physics (Kenichi ISHIKAWA) for internal use only (Univ. of Tokyo) This lecture is recorded for possible on-demand streaming. Chat your student ID number and full name. メスバウアー効果 ドップラーシフト Mössbauer effect + Doppler shift 一般相対性理論の検証 Test of Albert Einstein's theory of general relativity by Pound and Rebka, 1959 Gravitational red shift of light Clocks run differently at different places in a gravitational field Gravitational shift gamma ray (14.4 keV) 57Fe $h(f_r - f_e) = mgH$ $hf_e = mc^2$ $\frac{f_r}{f_e} = 1 + \frac{gH}{c^2}$ Ι = 22.5Doppler shift З Jefferson Laboratory $\frac{f_r}{f_e} = \sqrt{\frac{1 - v/c}{1 + v/c}} \approx 1 - \frac{v}{c}$ (Harvard University) f_r https://en.wikipedia.org/wiki/ Pound%E2%80%93Rebka experiment $v = \frac{gH}{c} = 7.36 \times 10^{-7} \,\mathrm{m/s}$ blue shift 57**Fe** by falling

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Weak interaction and beta decay 弱い相互作用とベータ壊変(ベータ崩壊)

Four fundamental interactions

interaction 相互作用	exchanged particle (gauge boson)	decay 壞変
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https://www.slideshare.net/yschhabra/radioactivity-45823825

Emitted electron (positron) energy has a broad distribution



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The existence of the neutrino was predicted by Wolfgang Pauli in 1930 to explain how beta decay could conserve energy, momentum, and angular momentum.

Pauli

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fundamental processes



cf. $m_{pion} = 139.570 \text{ MeV/c}^2$ (±), 134.9766 MeV/c² (neutral)

By transforming the Feynman diagram ...



n	$\rightarrow p e^{-}$	$\bar{ u}_e$	beta-
	-	-	

- $p \rightarrow n e^+ \nu_e$ beta+
- $pe^- \rightarrow n\nu_e$ electron capture (EC)
- $\bar{\nu}_e \, \mathrm{p} \to \mathrm{e}^+ \, \mathrm{n}$ neutrino detection

Fermi theory of beta decay

Decay rate $w = \frac{2\pi}{\hbar} \left| \langle \psi_{\rm p} \psi_{\rm e} | H_{\beta} | \psi_{\rm n} \psi_{\nu} \rangle \right|^2 \frac{dn}{dE}$ Fermi's golden rule density of state 状態密度 $\approx \int e^{-ik_{\mathrm{p}}\mathbf{r}_{2}} e^{-ik_{\mathrm{e}}\mathbf{r}_{2}} H_{\beta} \left(\mathbf{r}_{2}-\mathbf{r}_{1}\right) e^{ik_{\mathrm{n}}\mathbf{r}_{1}} e^{ik_{\nu}\mathbf{r}_{1}} dV$ weak interaction is a short-range force $H_{\beta}(\mathbf{r}_2 - \mathbf{r}_1) \sim G\delta(\mathbf{r}_2 - \mathbf{r}_1)$ $\approx G$ Electron energy distribution dominated by density of state 放出される電子のエネルギー分布は状態密度で 決まる

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Density of state 状態密度 assuming plane waves p : electron momentum $dn \propto p^2 dp q^2 dq$ q: neutrino momentum energy $Q = E_e + E_{\nu}$ $E_{\nu} = cq$ $E_e = \sqrt{m_e^2 c^4 + p^2 c^2}$ electron neutrino dn $q \propto p^2 q^2 dp \propto \left(Q - E_e\right)^2 p^2 dp$ statistical factor 統計因子 $dE = dE_{\nu} = cdq$ Experiment ⁶⁴Cu β^+ Experiment Q = 0.653 keVTheory n(p) (arb. unit) Coulomb repulsion 0 0 \cap 1.0 1.5 0.0 0.5 2.0 p/m_ec

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β^+ decay and electron capture



Both may not always be energetically possible!

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Symmetry and conservation law

対称性と保存則

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— no change under a transformation

Any symmetry of a physical law has a corresponding conservation law

Noether's theorem ネーターの定理

symmetry	conserved quantity
temporal translation	energy
spatial translation 平行移動	momentum
rotation 回転	angular momentum
reflection r→-r (P) 空間反転	parity
time reversal (T) 時間反転	T-parity
charge conjugation (C) 粒子反粒子変換	C-parity
gauge invariance ゲージ不変性	electric charge



https://ja.wikipedia.org/wiki/エミー・ネーター

 $-{f r}_2|^2$

Example: Coulomb force

$$V(\mathbf{r}) = \frac{q_1 q_2}{4\pi\epsilon_0 |\mathbf{r}|^2} \quad \text{or} \quad V(\mathbf{r}_1, \mathbf{r}_2) = \frac{q_1 q_2}{4\pi\epsilon_0 |\mathbf{r}_1|^2}$$

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example in the classical mechanics

Hamilton equations

$$\dot{q}_i = \frac{\partial H}{\partial p_i} \qquad \dot{p}_i = -\frac{\partial H}{\partial q_i}$$

If the Hamiltonian does not explicitly depend on q_i (invariant under the spatial translation)





gauge invariance ゲージ不変性

$$\mathbf{B} =
abla imes \mathbf{A}, \quad \mathbf{E} = -rac{\partial \mathbf{A}}{\partial t} -
abla \phi$$

A

invariant under the gauge transformation

$$\mathbf{A} \to \mathbf{A}' = \mathbf{A} + \nabla \chi, \quad \phi' = \phi - \frac{\partial \chi}{\partial t}$$

Invariance of the Action S 作用素積分

Conservation of the electric charge

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = 0$$
₃₃

reflection

 $\hat{\pi}\psi(\mathbf{r}) = \psi(-\mathbf{r})$ $\hat{\pi}^{2}\psi(\mathbf{r}) = \psi(\mathbf{r})$ $\hat{\pi}^{2}\psi(\mathbf{r}) = \psi(\mathbf{r})$ Eigenvalues $\rightarrow \pm 1$

If the physical law is invariant under the reflection (gravitational, electromagnetic, and strong interaction)

Heisenberg's equation of motion

$$i\hbar\frac{d\hat{\pi}}{dt} = [\hat{\pi}, H] = 0$$

Conservation of parity

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parity violation パリティ非保存 nonconservation of parity

in the weak interaction

- Prediction by T.-D. Lee and C. N. Yang in 1956
- Experimental verification by C.S.Wu in 1957



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CP violation



https://en.wikipedia.org/wiki/Makoto_Kobayashi_(physicist)

https://en.wikipedia.org/wiki/Toshihide_Maskawa

Makoto Kobayashi

Toshihide Maskawa

Nobel prize in physics (2008)



- Preservation of CPT symmetry by all physical phenomena
- Any Lorentz invariant local quantum field theory with a Hermitian Hamiltonian must have CPT symmetry