Fundamentals in Nuclear Physics
原子核基礎

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Schedule

4/27	Nuclear reactions
5/25	Nuclear decays and fundamental interactions

Report assignment (ITC-LMS) for each session

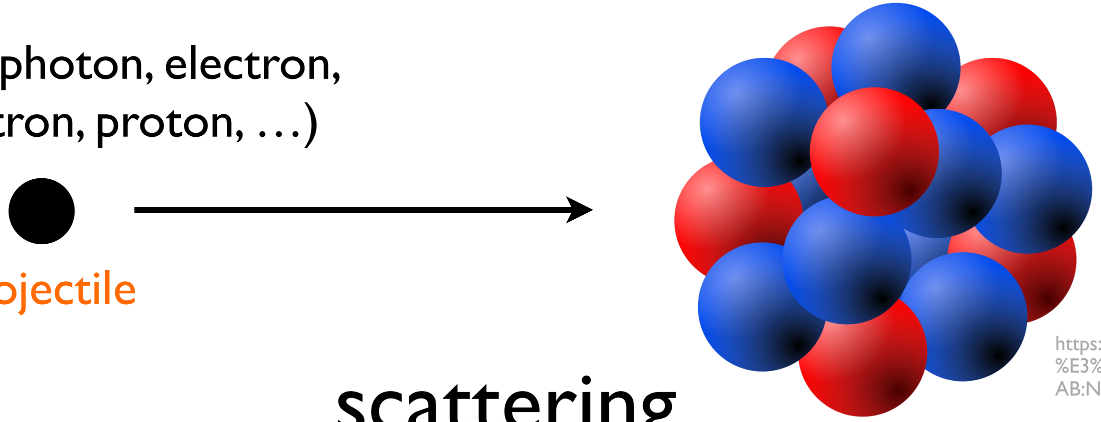
References

- Basdevant, Rich, and Spiro, "Fundamentals in Nuclear Physics" (Springer, 2005)
- Krane, "Introductory Nuclear Physics" (Wiley, 1987)
- 八木浩輔「原子核物理学」(朝倉書店, 1971)
- 石川顕一、高橋浩之「工学教程『原子核工学II』」（丸善、準備中）

Material downloadable from ITC-LMS and:

http://ishiken.free.fr/english/lecture.html

Nuclear reactions



free particle (photon, electron, positron, neutron, proton, …)

projectile

target

https://ja.m.wikipedia.org/wiki/%E3%83%95%E3%82%A1%E3%82%A4%E3%83%AB:Nucleus_drawing.svg

scattering

nuclear reactions

Examples

$\alpha + {}^{14}\mathrm{N} \rightarrow {}^{17}\mathrm{O} + p$ (Rutherford, 1919)

$p + {}^{7}\mathrm{Li} \rightarrow {}^{4}\mathrm{He} + \alpha$ (Cockcroft and Walton, 1930)

Typical nuclear reactions

$a + X \to Y + b$ ⇔ X(a,b)Y

projectile target

Important nuclear reactions for thermal energy generation

- Fission（核分裂）
- Fusion（核融合）

Energetics エネルギー論

a + X → Y + b

$$m_X c^2 + T_X + m_a c^2 + T_a = m_Y c^2 + T_Y + m_b c^2 + T_b$$

rest mass ↑ ($m_X c^2$)

kinetic energy ↑ (T_X)

reaction Q value

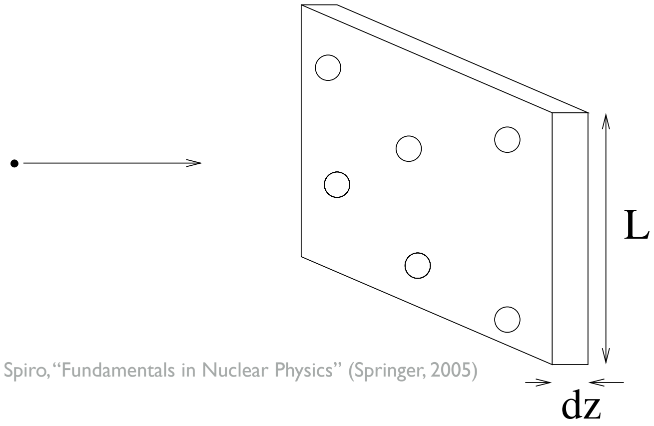
$$\begin{aligned} Q &= (m_{\text{initial}} - m_{\text{final}})c^2 \\ &= (m_X + m_a - m_Y - m_b)c^2 \\ &= T_Y + T_b - T_X - T_a \end{aligned}$$

excess kinetic energy

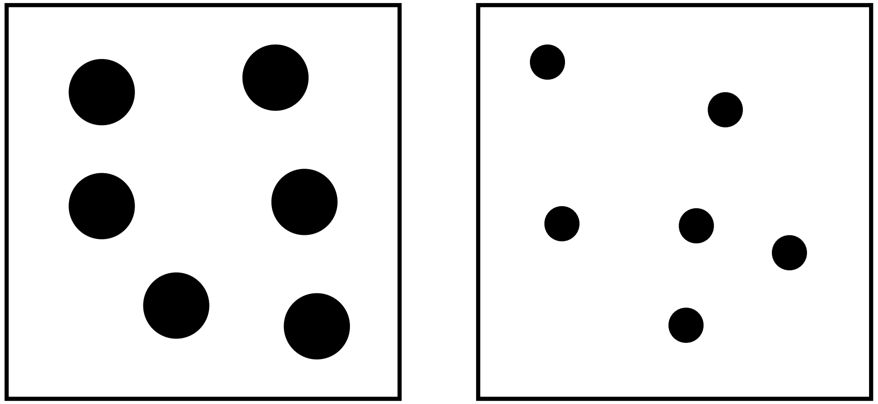
Q > 0 : exothermic 発熱反応

Q < 0 : endothermic 吸熱反応

Cross section 断面積



Basdevant, Rich, and Spiro, "Fundamentals in Nuclear Physics" (Springer, 2005)



number density n

radius r

reaction probability $\dfrac{(\pi r^2)n(L^2 dz)}{L^2} = \underline{\pi r^2} n dz$

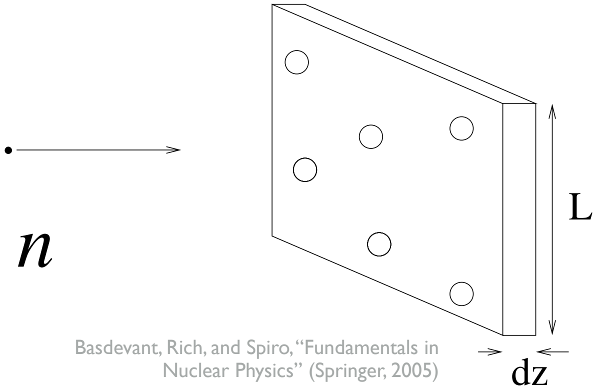
$\sigma \equiv \pi r^2$
"cross section"

$$\boxed{dP = \sigma n dz}$$

"Cross section" can be used to define a probability for any type of reaction

Probability P proportional to

- number density of target particles n
- target thickness dz



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$$dP = \sigma n dz$$

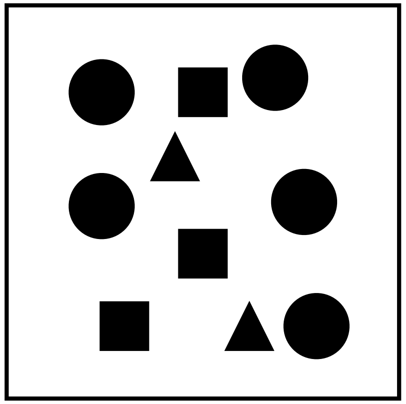
Unit of cross section

dimension of area $\longrightarrow$ m^2, cm^2

size of nucleus ~ a few fm

$\longrightarrow$ 1 barn (b) = 10^{-28} m^2 = 10^{-24} cm^2

Different types of target objects



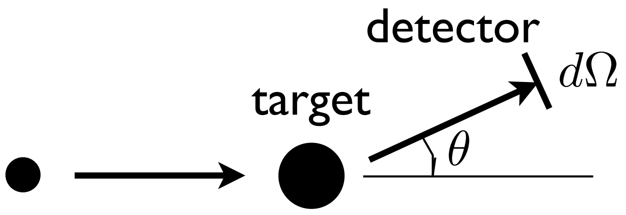
number density n_i

cross section σ_i

$$dP = dz \sum_i \sigma_i n_i$$

Differential cross section

angular dependence (角度依存性を考える)



Probability that the incident particle is scattered to a solid angle $d\Omega$

$$dP_{\theta,\phi} = \underline{\frac{d\sigma}{d\Omega}} n dz d\Omega$$

differential cross section
(微分断面積)

for isotropic scattering (等方散乱)

$$\frac{d\sigma}{d\Omega} = \frac{\sigma}{4\pi}$$

total cross section

$$\sigma = \int d\Omega \frac{d\sigma}{d\Omega} = \int_0^{2\pi} d\phi \int_0^{\pi} \frac{d\sigma}{d\Omega}(\theta, \phi) \sin\theta d\theta$$

Differential cross section

reaction creating *N* particles $\quad$ a b ⟶ $x_1\ x_2\ x_3\ \ldots\ x_N$

probability to create the particles xi in the momentum ranges $d^3\mathbf{p}_i$ around $\mathbf{p}_i$

$$dP = \frac{d\sigma}{d^3\mathbf{p}_1 \cdots d^3\mathbf{p}_N} n_b dz\, d^3\mathbf{p}_1 \cdots d^3\mathbf{p}_N$$

differential cross section (微分断面積)

total probability for the reaction $\quad dP_{ab\to x_1\cdots x_N} = \sigma_{ab\to x_1\cdots x_N} n_b dz$

reaction cross section

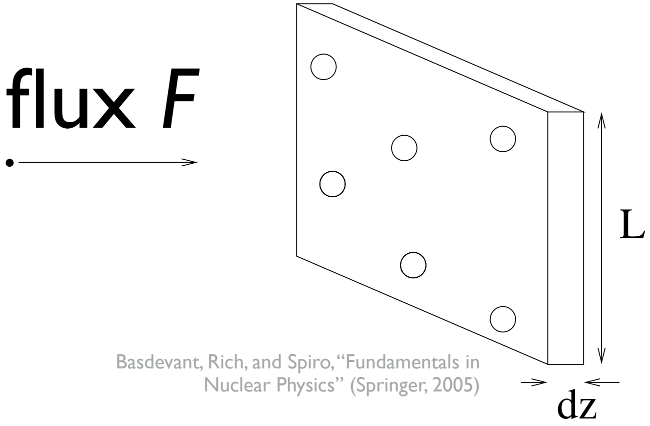
$$\sigma_{ab\to x_1\cdots x_N} = \int d^3\mathbf{p}_1 \cdots \int d^3\mathbf{p}_N \frac{d\sigma}{d^3\mathbf{p}_1 \cdots d^3\mathbf{p}_N} d^3\mathbf{p}_1 \cdots d^3\mathbf{p}_N$$

if there are more than one reactions

$$dP = \sigma_{\rm tot} n_b dz \qquad \sigma_{\rm tot} = \sum_i \sigma_i$$

平均自由行程　　　反応速度

Mean free path and reaction rate



Basdevant, Rich, and Spiro, "Fundamentals in Nuclear Physics" (Springer, 2005)

$$dF = -F\sigma n dz$$

$$\frac{dF}{dz} = -F\sigma n$$

$$F(z) = F(0)e^{-\sigma n z} = F(0)e^{-\Sigma z}$$

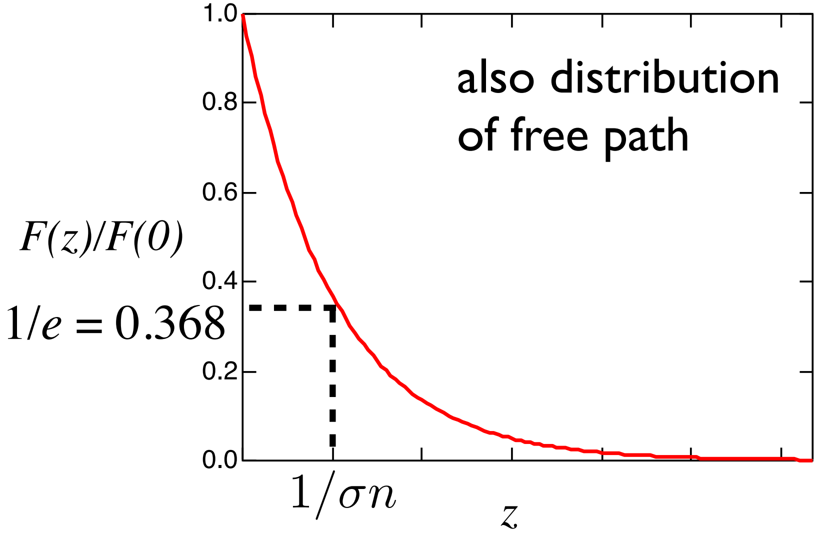
macroscopic cross section (マクロ断面積)　$\Sigma = \sigma n$　[1/length]

mean free path　$l = 1/\sigma n$

if there are different types of target objects (nuclei)

$$l = 1/\sum_i \sigma_i n_i$$

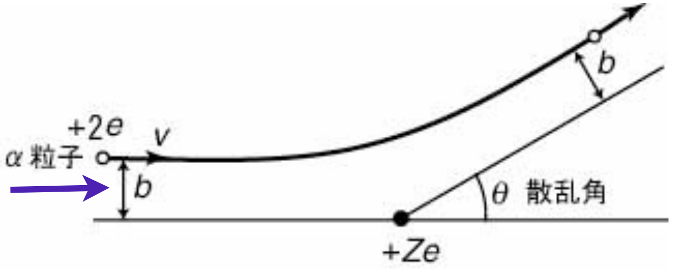
reaction rate　$\frac{v}{l} = n\,\sigma v$



also distribution of free path

differential cross section of scattering in general

衝突係数
impact parameter →



量子力学(小出昭一郎 ,裳華房)

classical scattering in general

$$b \longleftrightarrow \theta(b)$$

$$b+db \longleftrightarrow \theta(b+db) = \theta + d\theta = \theta(b) + \frac{d\theta}{db}db$$

$$d\sigma = 2\pi b db \longleftrightarrow d\Omega = -2\pi \sin\theta d\theta$$

$$\boxed{\frac{d\sigma}{d\Omega} = \left|\frac{b(\theta)}{\sin\theta}\frac{db}{d\theta}\right|}$$

Example: hard sphere with a radius *R*

$$b = R\cos\frac{\theta}{2} \quad \Rightarrow \quad \frac{d\sigma}{d\Omega} = \frac{R^2}{4} \qquad \sigma = \pi R^2$$

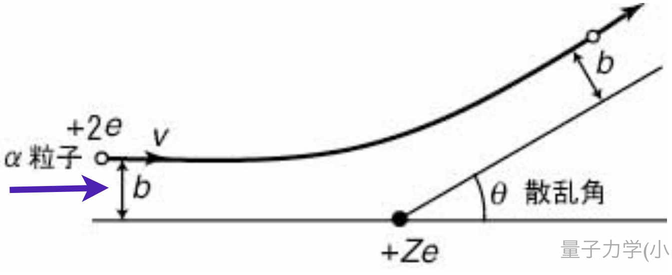
geometrical cross section

ラザフォード散乱

Rutherford scattering

scattering of a charged particles by a Coulomb potential

衝突係数
impact parameter →



量子力学(小出昭一郎 ,裳華房)

$$V(r) = \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 r}$$

$$b = \frac{Z_1 Z_2 e^2}{8\pi\epsilon_0 E_k} \cot\frac{\theta}{2} \quad \Rightarrow \quad \frac{d\sigma}{d\Omega} = \left(\frac{Z_1 Z_2 e^2}{16\pi\epsilon_0 E_k}\right)^2 \frac{1}{\sin^4\frac{\theta}{2}}$$

The same result is obtained by the quantum theory.

$$\sigma = \infty$$

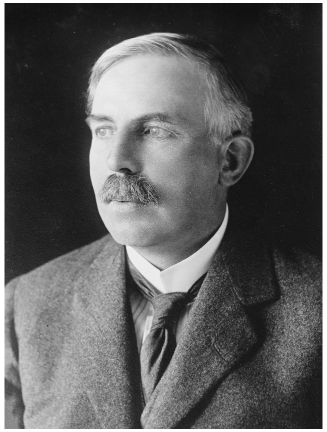
Coulomb force is long-range　長距離力

- Incident particle is scattered no matter how large the impact parameter may be.
- Practically, the Coulomb potential is screened at large distances by oppositely charged particles

Rutherford scattering

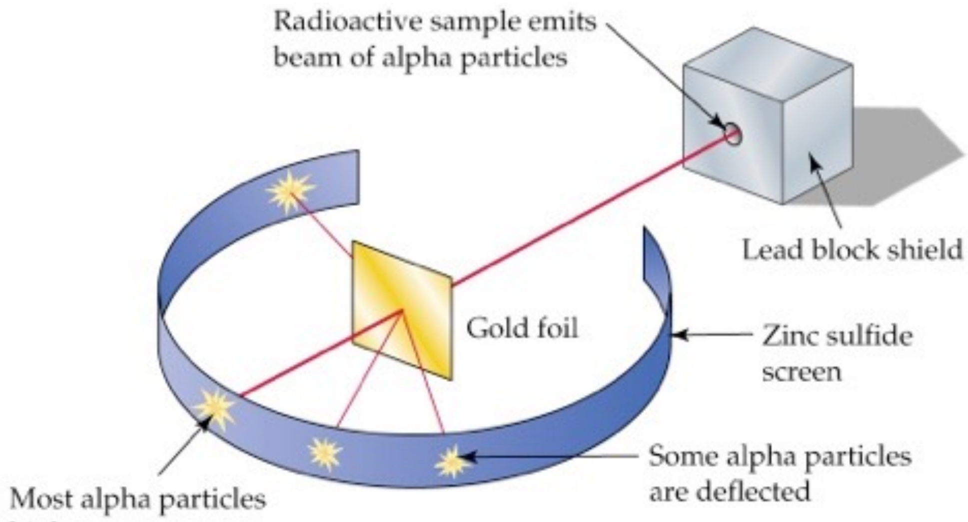
Geiger-Marsden experiment (1909) ガイガー・マースデンの実験

Ernest Rutherford



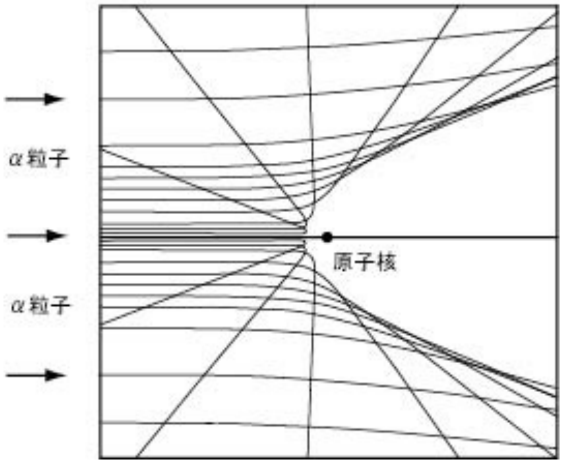
1871〜1937

https://ja.wikipedia.org/wiki/アーネスト・ラザフォード

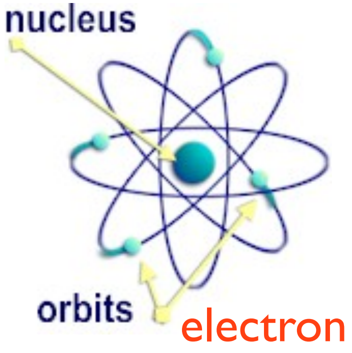


(a) http://mishiewishieblogg.blogspot.jp/

α粒子散乱の軌道

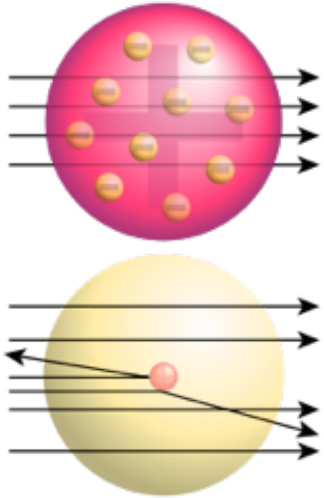


量子力学(小出昭一郎 ,裳華房)



1911

Rutherford (or planetary) model



General characteristics of cross-sections

Elastic scattering 弾性散乱

The internal states of the projectile and target (scatterer) do not change before and after the scattering.

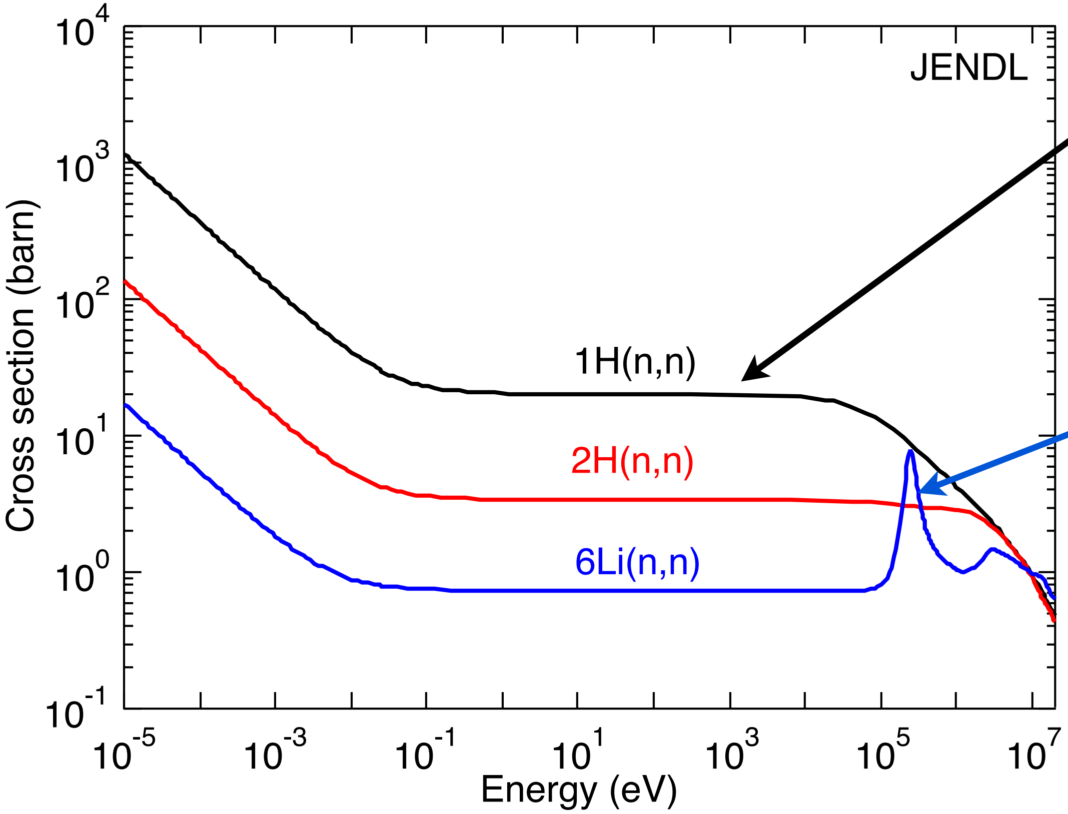
- Rutherford scattering, (n,n), (p,p), etc.

Inelastic scattering 非弾性散乱

- (n,γ), (p,γ), (n,α), (n,p), (n,d), (n,t), etc.
- fission, fusion

Elastic neutron scattering

- relevant to (neutron) moderator in nuclear reactors 中性子減速材
- due to the short-range strong interaction



range of the strong interaction

flat region

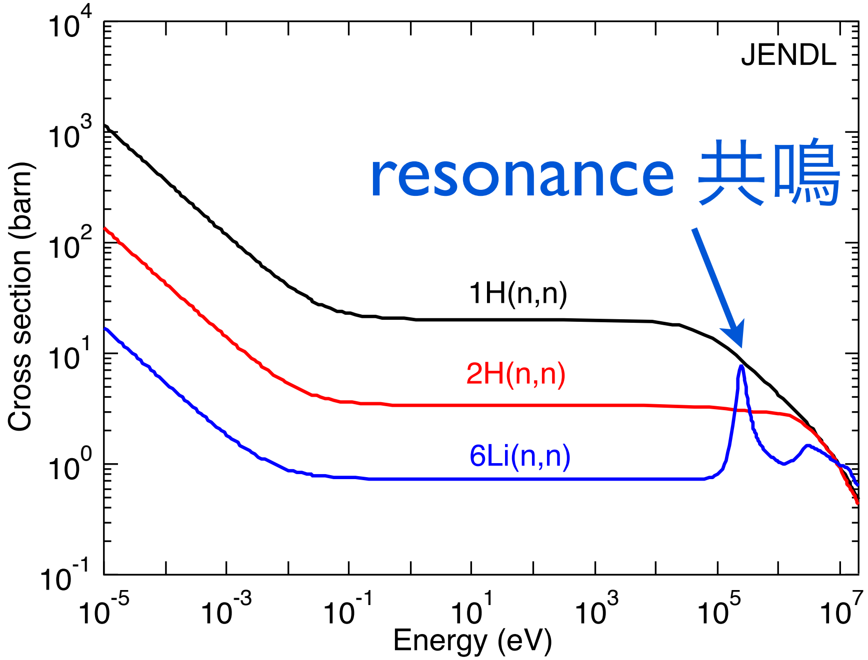
$$\sigma_{el} \sim 20\,\mathrm{b} \gg \pi(2\mathrm{fm})^2 \sim 0.1\,\mathrm{b}$$

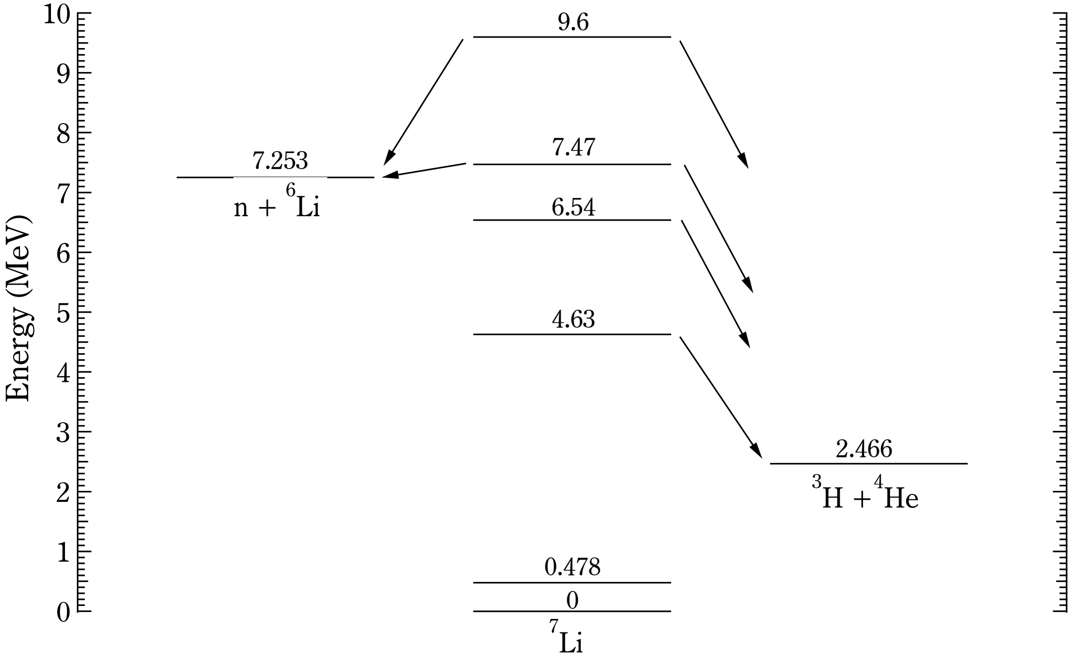
resonance

$$p > h/r$$

$$(p^2/2m_n > 200\,\mathrm{MeV})$$

Elastic neutron scattering





The energy levels of ^{7}Li and two dissociated states n-^{6}Li and ^{3}H-^{4}He (t-α)

$$n + {}^{6}\mathrm{Li} \rightarrow {}^{7}\mathrm{Li}^{*} \rightarrow n + {}^{6}\mathrm{Li}$$

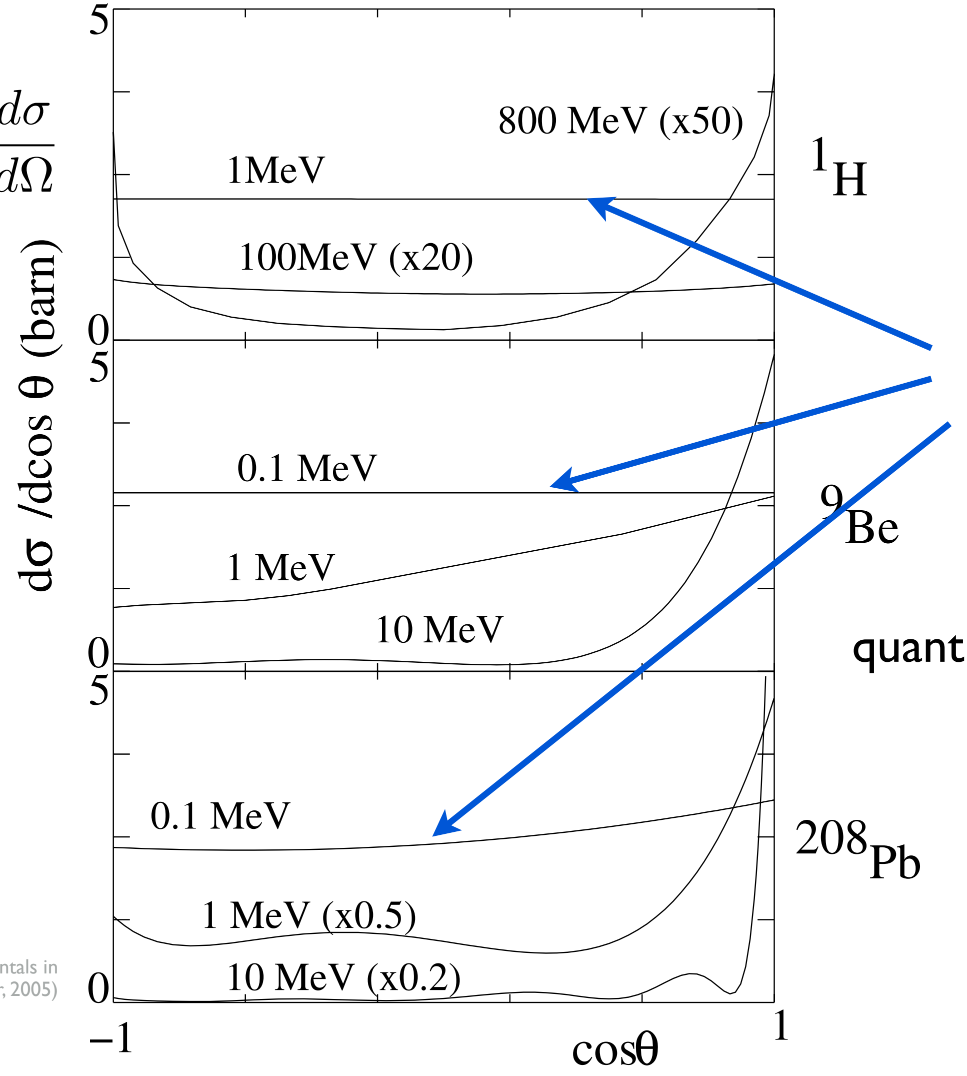
Nuclear data libraries

- ENDF (Evaluated Nuclear Data File, USA)
- JENDL (Japanese Evaluated Nuclear Data Library, Japan)
- JEFF (Joint Evaluated Fission and Fusion file, Europe)
- CENDL (Chinese Evaluated Nuclear Data Library, China)
- ROSFOND (Russia)
- BROND (Russia)

http://www-nds.iaea.org/exfor/endf.htm

Differential cross section for elastic neutron scattering

$$\frac{d\sigma}{d\cos\theta} = 2\pi\frac{d\sigma}{d\Omega}$$



Basdevant, Rich, and Spiro, "Fundamentals in Nuclear Physics" (Springer, 2005)

isotropic for

$$p < \frac{\hbar}{R_{\mathrm{nucleus}}}$$

quantization of angular momentum

$$L = pR_{\mathrm{nucleus}} < \hbar$$

⬇

s-wave scattering

Inelastic scattering

中性子捕獲反応

Neutron capture

neutron binding energy = ca. 8 MeV

↓

発熱反応

exothermic reaction in most cases

放射化

activation

Highly excited states formed, which subsequently decay.

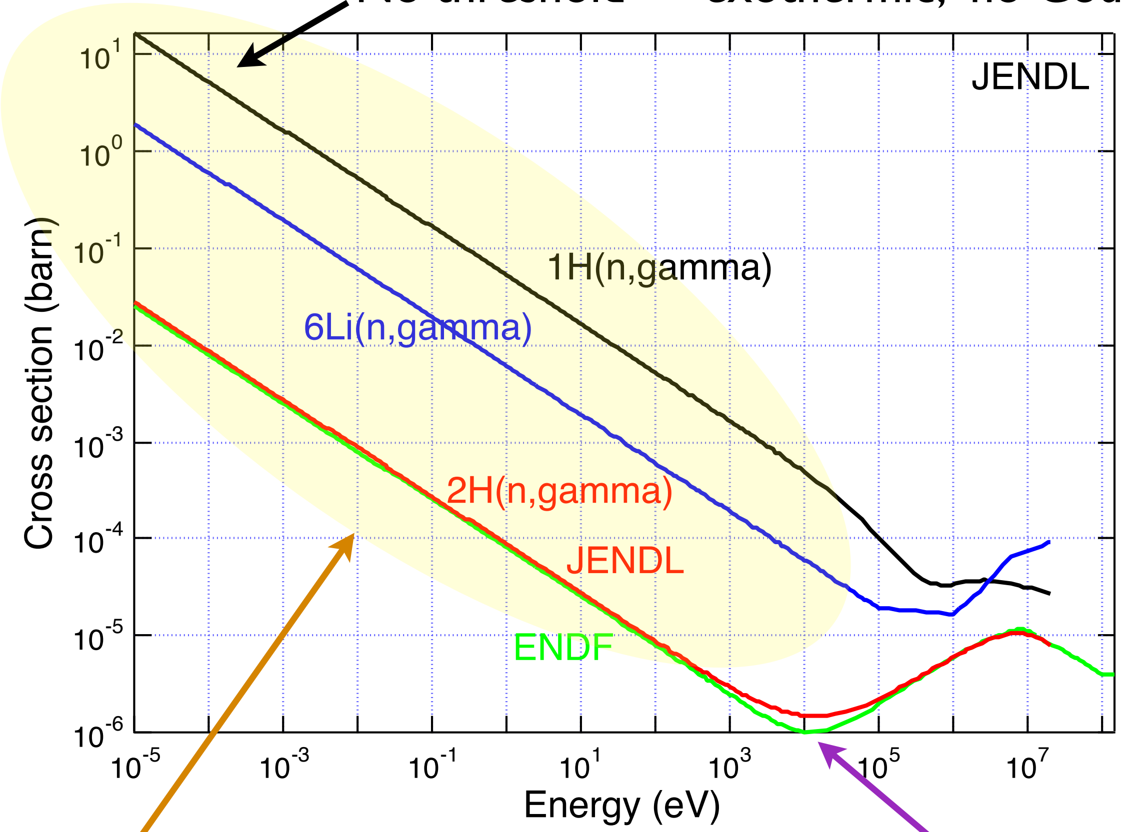
- Radiative capture 放射捕獲（放射性捕獲） $^{A}X(n,\gamma)^{A+1}X$
  - emits a gamma ray
  - $^{113}Cd(n,\gamma)^{114}Cd$ ← neutron shield
- Other neutron capture reactions
  - $^{10}B(n,\alpha)^{7}Li$, $^{3}He(n,p)^{3}H$, $^{6}Li(n,t)^{4}He$
  - Applications: neutron detector, shield, neutron capture therapy for cancer

Inelastic scattering

放射捕獲（放射性捕獲）

neutron radiative capture

No threshold ← exothermic, no Coulomb barrier



discrepancy between JENDL and ENDF

$$\sigma \propto E^{-1/2} \propto 1/v$$

$\boxed{1/v \text{ law}}$ ➡ Energy-independent reaction rate σv

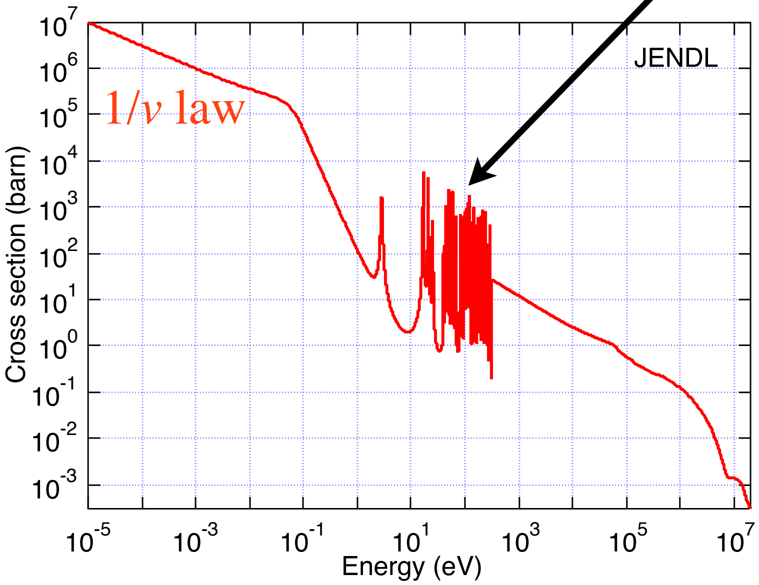
Neutron capture reactions with large cross section

- ^{113}Cd(n,γ)^{114}Cd : shield
- ^{157}Gd(n,γ)^{158}Gd : neutron absorber in nuclear fuel, cancer therapy
- ^{10}B(n,α)^{7}Li : detector, cancer therapy
- ^{3}He(n,p)^{3}H : detector
- ^{6}Li(n,t)^{4}He : shield, filter, detector

Inelastic scattering

^{157}Gd(n,γ)^{158}Gd

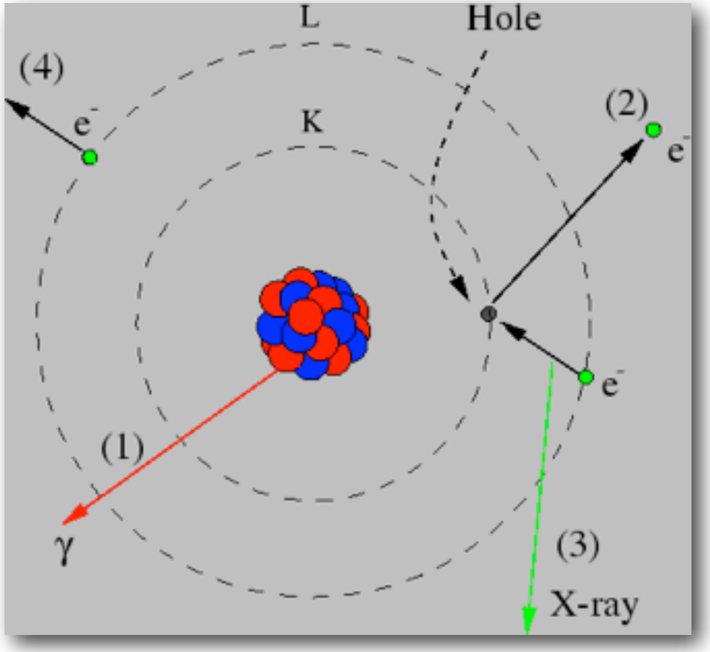
Heavy nuclei have many excited states.
→ Complicated resonance structure



thermal neutron
= 0.025 eV

オージェ効果
(4) Auger effect

内部転換
(2) internal conversion



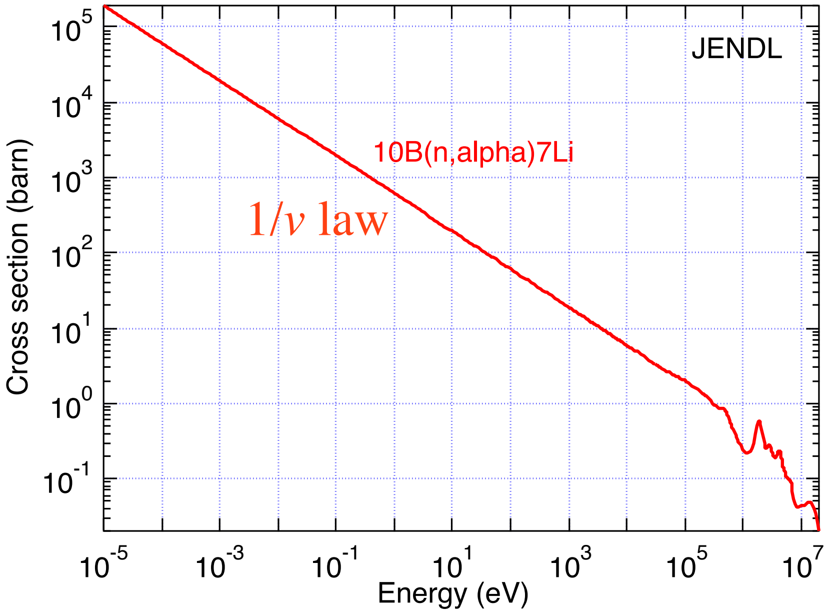
http://www.nuclear.kth.se/courses/lab/latex/internal/internal.html

Possible channels
- (1)
- (2)+(3)
- (2)+(4)

Applications
- Burnable poison Gd_2O_3 (neutron absorber in nuclear fuel)
- Gadolinium neutron capture therapy (GdNCT) for cancer

Inelastic scattering

$^{10}B(n,\alpha)^7Li$



Applications

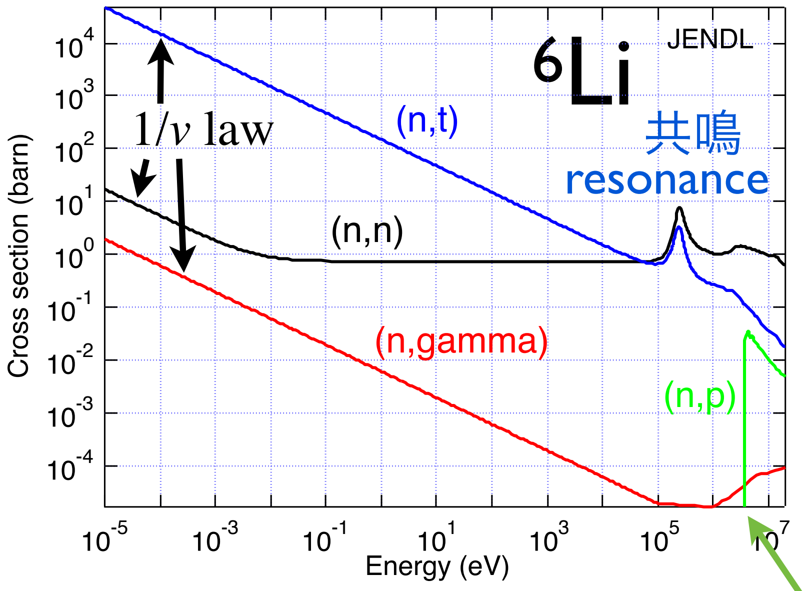
- BF_3 proportional counter
- Boron neutron capture therapy (BNCT) for cancer

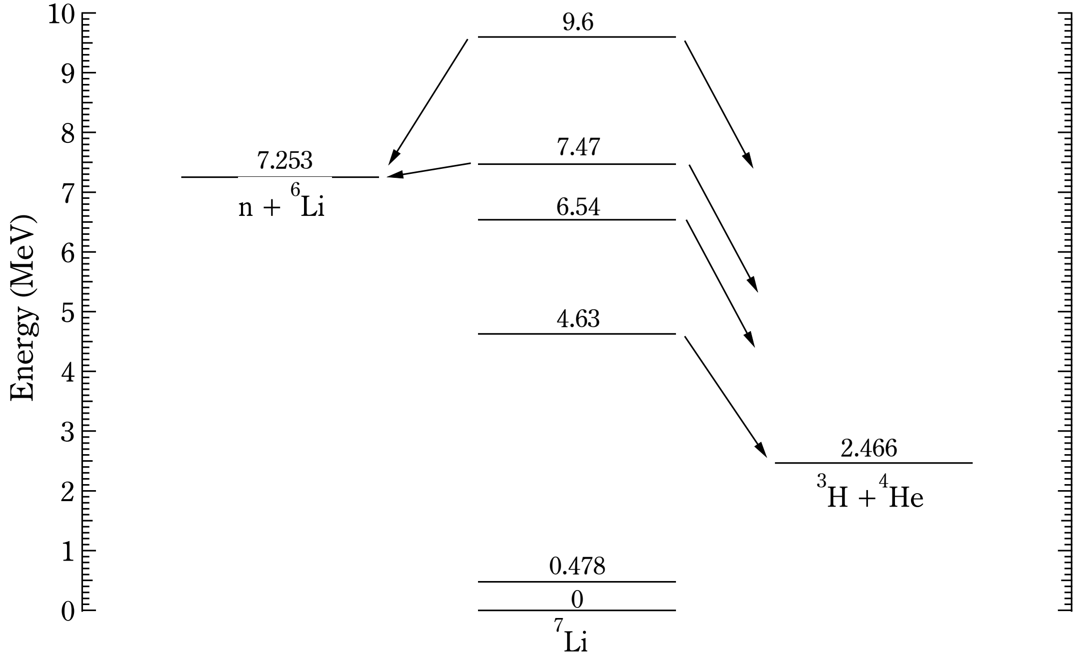
$^3He(n,p)^3H$

- Helium-3 proportional counter

Inelastic scattering

Neutron capture by ^{6}Li





The energy levels of ^{7}Li and two dissociated states n-^{6}Li and ^{3}H-^{4}He (t-α)

^{6}Li(n,t)^{4}He

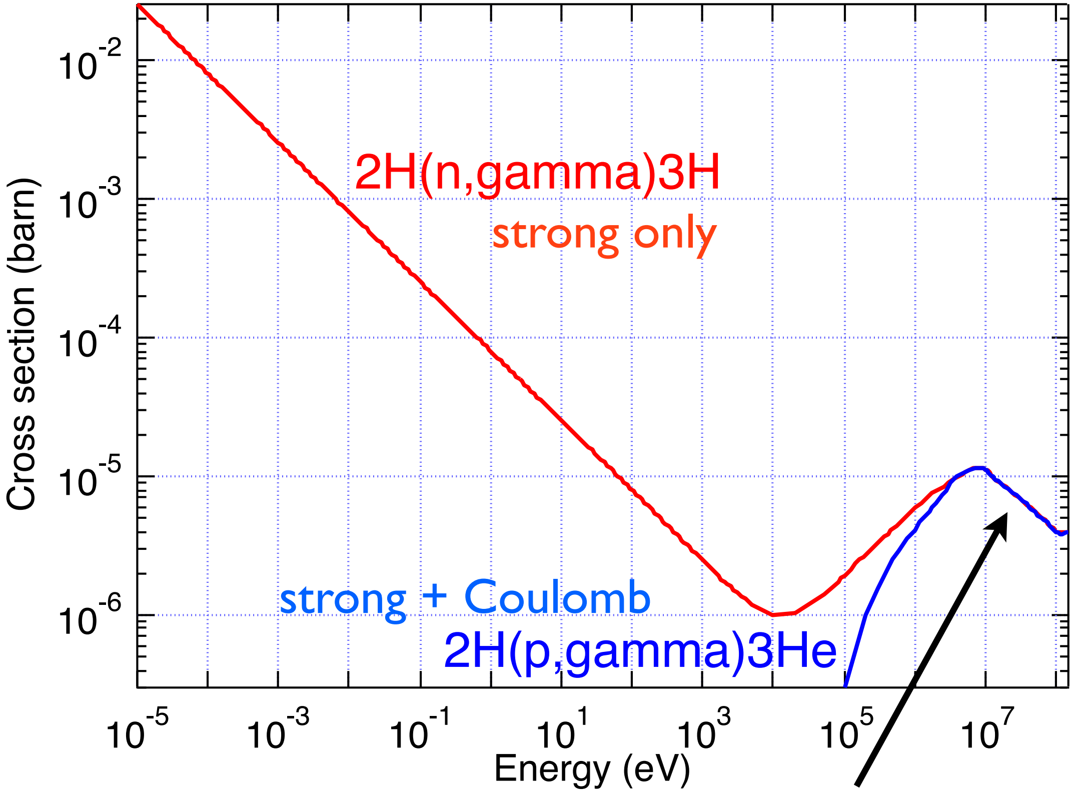
- LiF neutron shield and filter
- Neutron detector

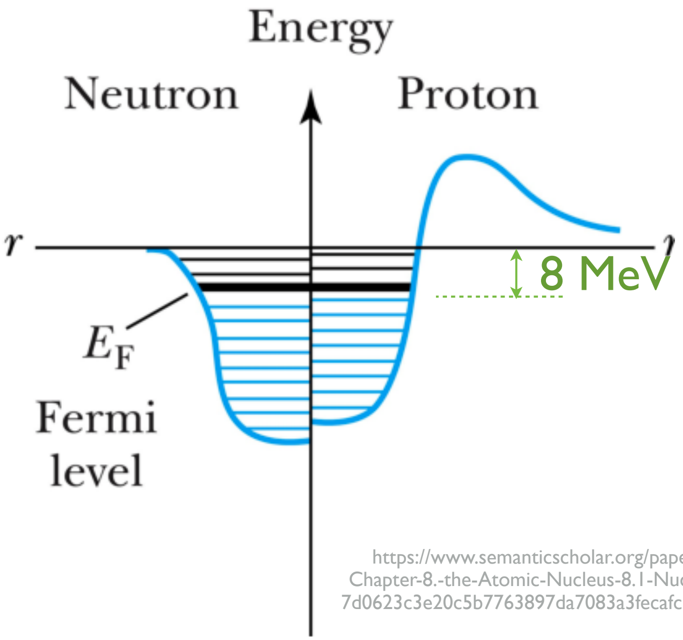
threshold ← endothermic reaction
しきい値 吸熱反応

Inelastic proton scattering

Coulomb barrier

The low-energy cross-section for inelastic reactions are strongly affected (suppressed) by Coulomb barriers through which a particle must tunnel.





https://www.semanticscholar.org/paper/Chapter-8.-the-Atomic-Nucleus-8.1-Nuclear/7d0623c3e20c5b7763897da7083a3fecafc778e0

proton energy > the Coulomb potential energy at the ^{6}Li surface

$$\frac{3e^2}{4\pi\epsilon_0}\frac{1}{2.4\,\mathrm{fm}} \sim 1.8\,\mathrm{MeV}$$

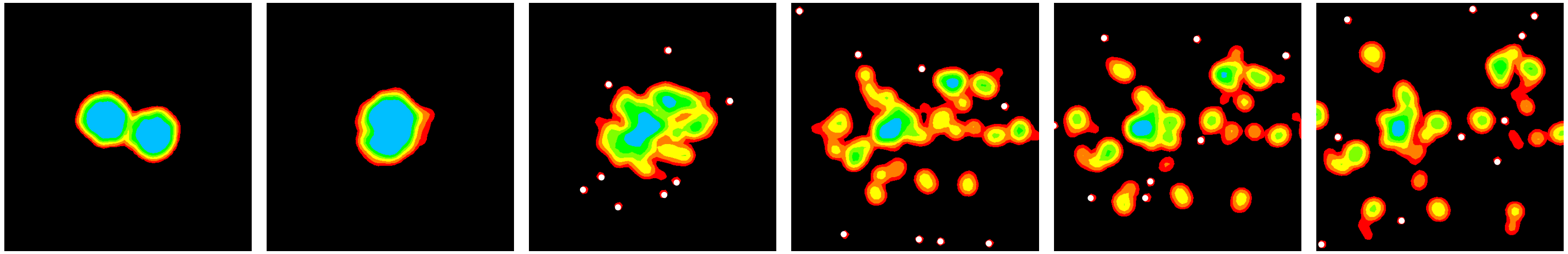
High-energy inelastic nucleus-nucleus collision

Coulomb barrier ineffective for $E_{cm} > \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 R}$ sum of the radii of the two nuclei

Energy < 1 GeV/u (GeV/nucleon)

Total inelastic cross section ~ order of πR^2

Break up of one or both of the nuclei



- Fragmentation reaction - for medium-*A* nuclei 核破砕反応
- Collision-induced fission - for heavy nuclei
- Spallation - fragmentation by protons or neutrons

- application: production of unstable (radioactive) nuclides
- issue in carbon-ion cancer therapy

Fusion evaporation reaction　核融合蒸発反応

- Occasionally, the target and projectile may fuse to form a much heavier nucleus.
- The produced excited nucleus emits neutrons until a bound nucleus is produced.
- used to produce trans-uranium elements　超ウラン元素

http://www.phy.ornl.gov/hribf/science/abc/fusion-evap.shtml

Energy > 1 GeV/u

- Production of pions and other hadrons

cosmic-ray protons ⟶ upper-atmosphere nuclei

$\Downarrow$

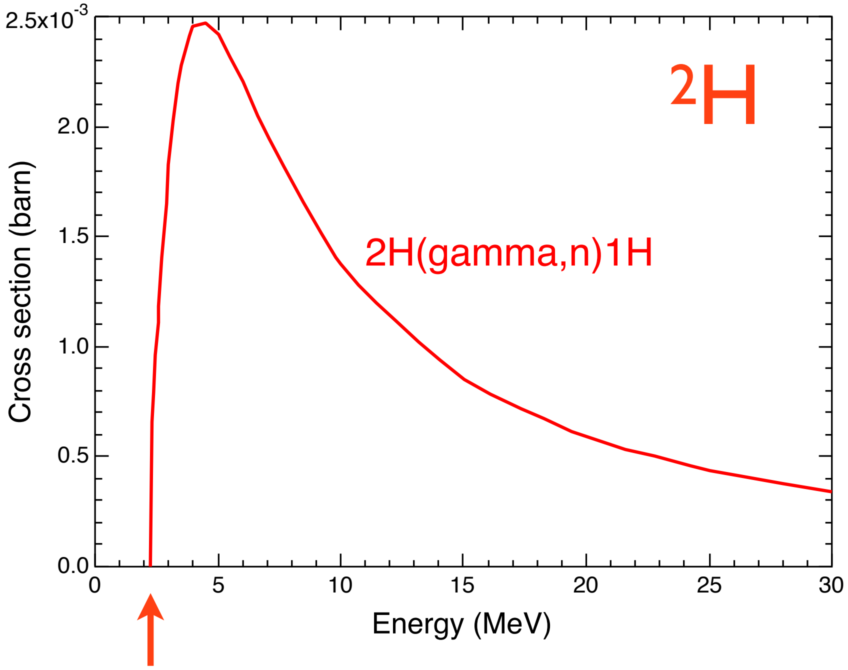
pions　$\pi^+ \rightarrow \mu^+ + \nu_\mu$　　$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$

muons - primary component of cosmic rays on the ground

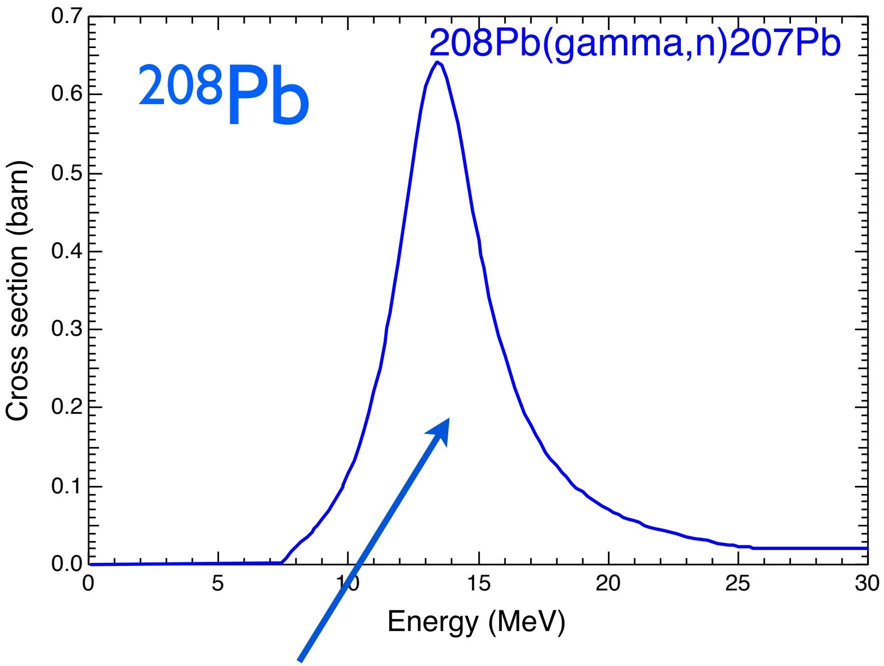
光核反応

Photo-nuclear reaction

- Excitation and break-up (dissociation) through photo-absorption
  - Analog of the photoelectric effect



threshold (2.22 MeV) = binding energy of ^{2}H



giant resonance 巨大共鳴

collective oscillation of protons in the nucleus

ニュートリノ

Neutrino reaction

- Only weak interactions
- Cross section ~ 10^{-48} m^2

$$\nu_e\, e^- \rightarrow \nu_e\, e^-$$

$$\bar{\nu}_e\, e^- \rightarrow \bar{\nu}_e\, e^-$$

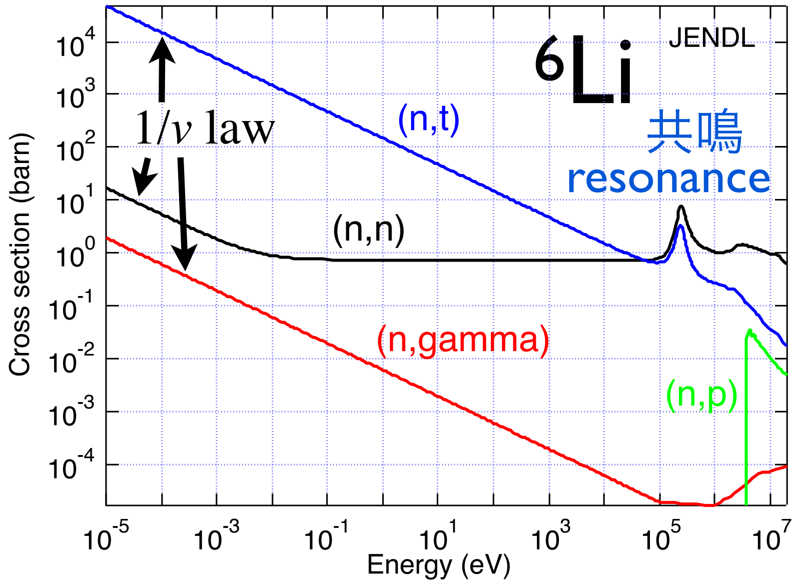
$$\nu_\mu\, e^- \rightarrow \nu_\mu\, e^-$$

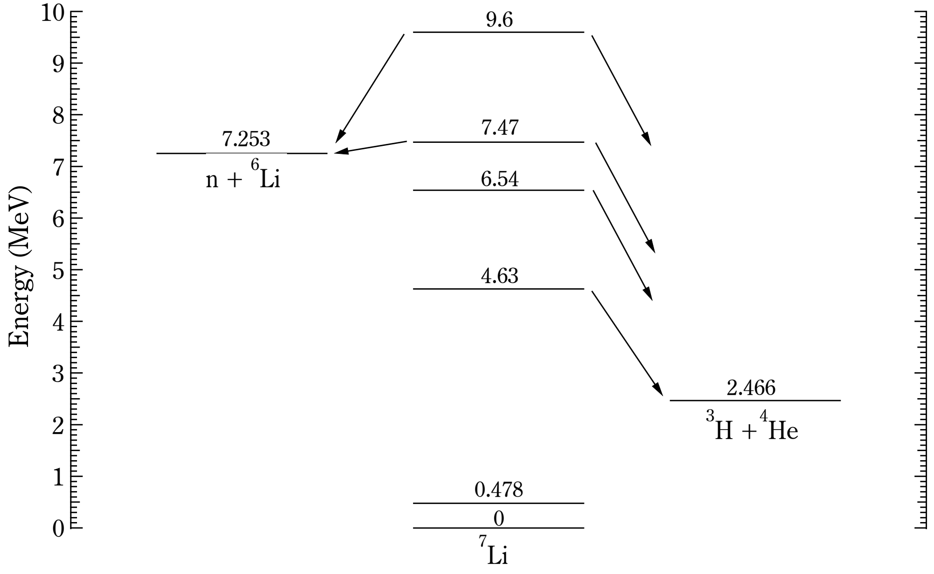
$$\bar{\nu}_\mu\, e^- \rightarrow \bar{\nu}_\mu\, e^-$$

$$\nu_e\, n \rightarrow e^-\, p$$

$$\bar{\nu}_e\, p \rightarrow e^+\, n$$

Resonance 共鳴



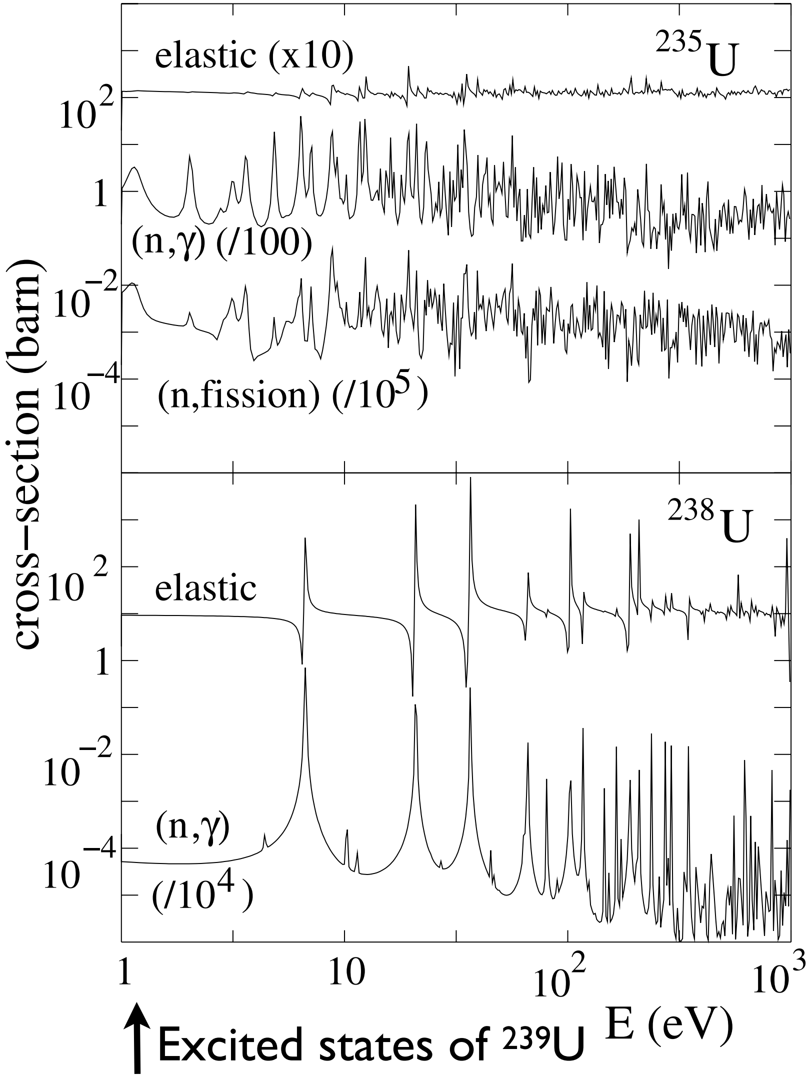


重い核には多くの励起状態

Many excited states for heavy nuclei

→ complicated resonance structure

複雑なエネルギー依存性



↑Excited states of ^{239}U

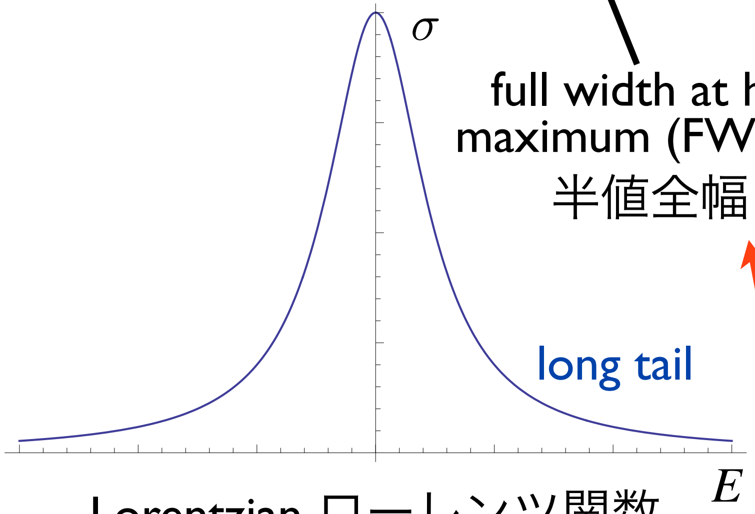
Basdevant, Rich, and Spiro, "Fundamentals in Nuclear Physics" (Springer, 2005)

Resonance line shape

Resonance

$$\sigma(E) \sim \frac{A}{(E-E_0)^2+(\Gamma/2)^2}$$

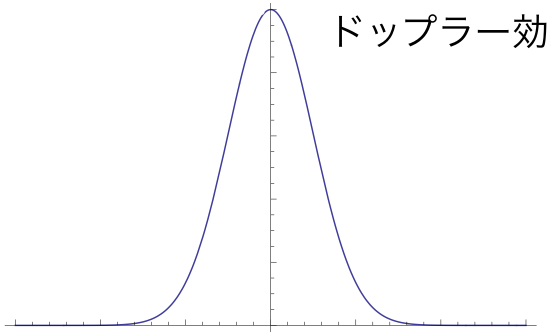
full width at half maximum (FWHM)
半値全幅



Lorentzian ローレンツ関数

Doppler effect

ドップラー効果



Life time $\tau = \hbar/\Gamma$

Decay rate $\tau^{-1} = \Gamma/\hbar$

$\Gamma \cdot \tau = \hbar \longleftarrow$ uncertainty principle
不確定性原理

Γ : 自然幅
natural width
homogeneous width

⟺

ドップラー幅
inhomogeneous width

$$\sim \exp\left[-\frac{(E-E_0)^2}{\Delta E^2}\right]$$

Time-dependent wave function of an excited state

$$\Psi(\mathbf{r},t) = \psi(\mathbf{r})e^{-iE_0t/\hbar} \Rightarrow |\Psi(\mathbf{r},t)|^2 = |\psi(\mathbf{r})|^2$$

does not decay

To be consistent with the exponential decay law

$$|\Psi(\mathbf{r},t)|^2 = |\psi(\mathbf{r})|^2 e^{-t/\tau}$$

$$\Rightarrow \Psi(\mathbf{r},t) = \psi(\mathbf{r})e^{-iE_0t/\hbar}e^{-t/2\tau}$$

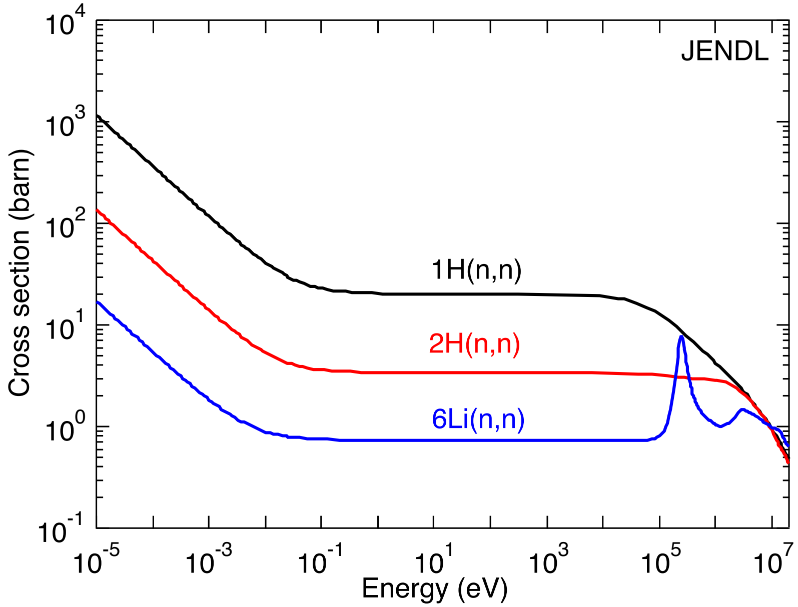
Energy spectrum (by Fourier transform)

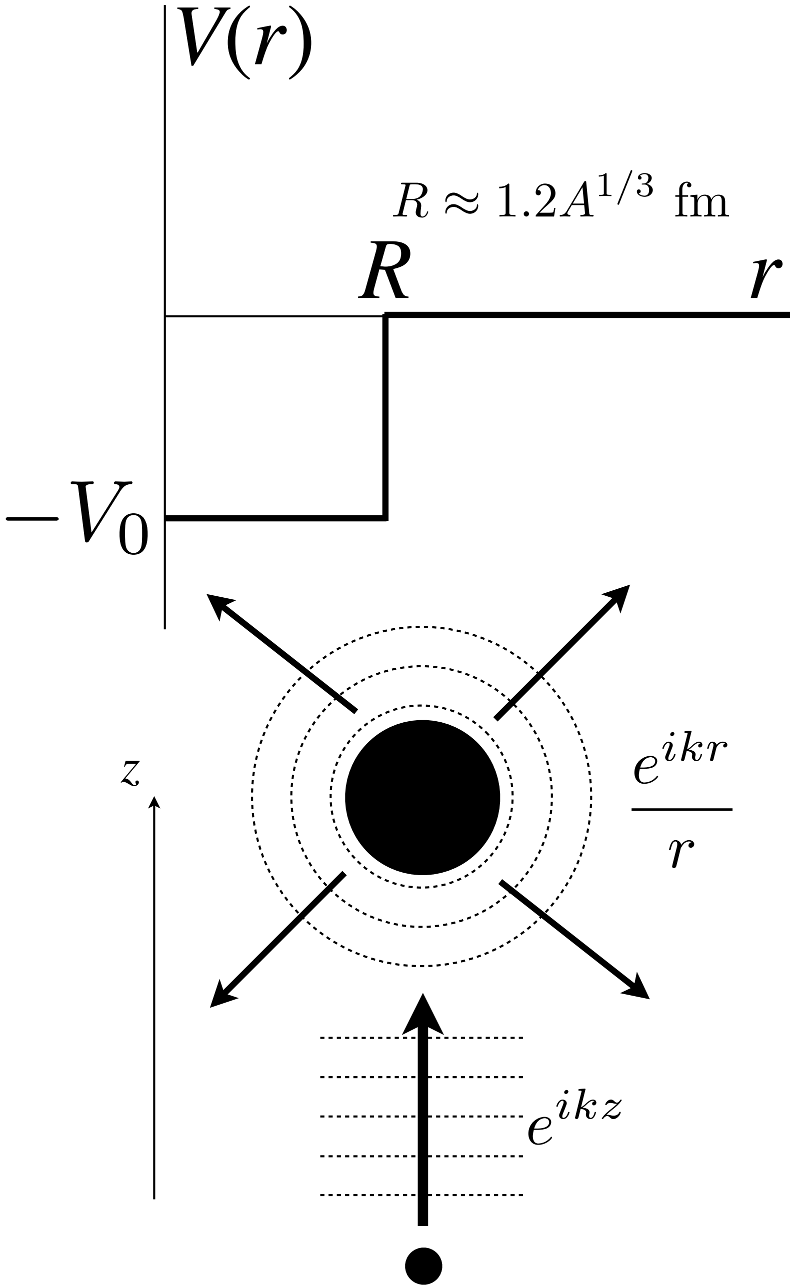
$$P(E) \propto \left| \int_0^\infty e^{iEt/\hbar}e^{-iE_0t/\hbar}e^{-t/2\tau}dt \right|^2$$

$$\propto \frac{1}{(E-E_0)^2 + (\Gamma/2)^2}$$

核子-原子核散乱の量子力学的取り扱い

Quantum treatment of nucleon-nucleus scattering





isotropic scattering 等方散乱

angular momentum 角運動量

$$L = \hbar k R \ll \hbar \quad \Longrightarrow \quad kR \ll 1$$

for neutron scattering 中性子散乱

$$E = \frac{p^2}{2m_n} \ll \frac{(\hbar c)^2}{2m_n c^2 R^2} \sim \frac{13\ \mathrm{MeV}}{A^{2/3}}$$

Schrödinger equation シュレーディンガー方程式

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V(r)\right)\psi_k(\mathbf{r}) = \frac{\hbar^2 k^2}{2m}\psi_k(\mathbf{r})$$

$$\psi_k(\mathbf{r}) = e^{ikz} + \frac{f e^{ikr}}{r} \qquad (r > R)$$

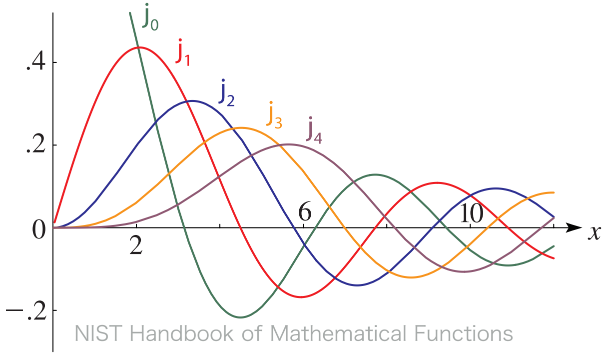
$$e^{ikz} = \sum_{l=0}^{\infty} (2l+1) i^l j_l(kr) P_l(\cos\theta)$$

spherical Bessel function

球ベッセル関数

Legendre polynomial

ルジャンドル多項式



NIST Handbook of Mathematical Functions

$$j_l(kr) \sim (kr)^l/(2l+1)!!$$

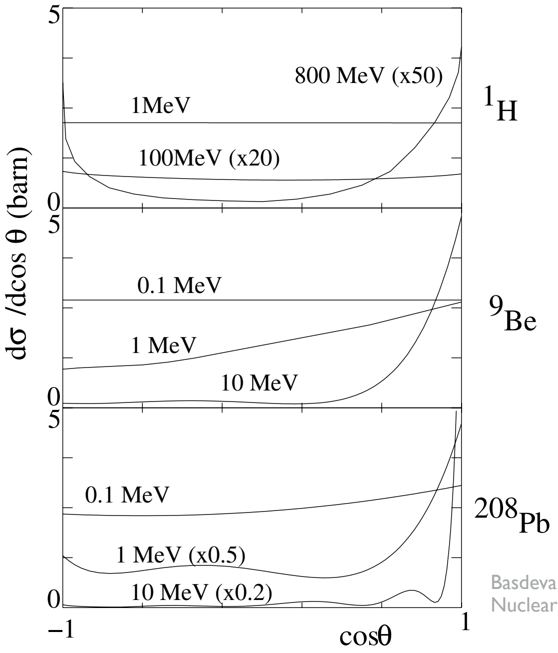
$$j_0(z) = \frac{\sin z}{z}$$

$$j_1(z) = \frac{\sin z}{z^2} - \frac{\cos z}{z}$$

$$P_0(\cos\theta) = 1$$

$$P_1(\cos\theta) = \cos\theta$$

$$P_2(\cos\theta) = \frac{3}{2}\cos^2\theta - \frac{1}{2}$$



Basdevant, Rich, and Spiro, "Fundamentals in Nuclear Physics" (Springer, 2005)

$$\psi_k(\mathbf{r}) = \underbrace{\left(e^{ikz} - \frac{\sin kr}{kr}\right)}_{\text{anisotropic}} + \underbrace{\left(\frac{\sin kr}{kr} + \frac{fe^{ikr}}{r}\right)}_{\text{isotropic}} \qquad (r > R)$$

anisotropic 非等方　　　isotropic 等方

$$\to \frac{\sin kr}{kr} + \frac{fe^{ikr}}{r}$$

isotropic

near the boundary　$kR \ll 1$

ポテンシャルの境界近く

➡ $\psi_k(\mathbf{r})$ isotropic also at $r < R$　ポテンシャル内でも等方

$$u_k(r) = r\,\psi_k(r)$$

➡ $$\left(-\frac{\hbar^2}{2m}\frac{d^2u_k}{dr^2} + V(r)\right)u_k(r) = \frac{\hbar^2k^2}{2m}u_k(r)$$

$$u_k(r) = \frac{\sin kr}{k} + fe^{ikr} \qquad (r > R)$$

Solution at $r < R$

$$u_k(r) = A \sin Kr \qquad\qquad (r < R)$$

Boundary condition

$u_k(r)$ and $u_k'(r)$ continuous at $r = R$

境界条件

$$kR \ll 1 \quad \Longrightarrow \quad f = R\left(\frac{\tan KR}{KR} - 1\right) \qquad K \approx \sqrt{\frac{2mV_0}{\hbar^2}}$$

low-energy scattering

Cross section

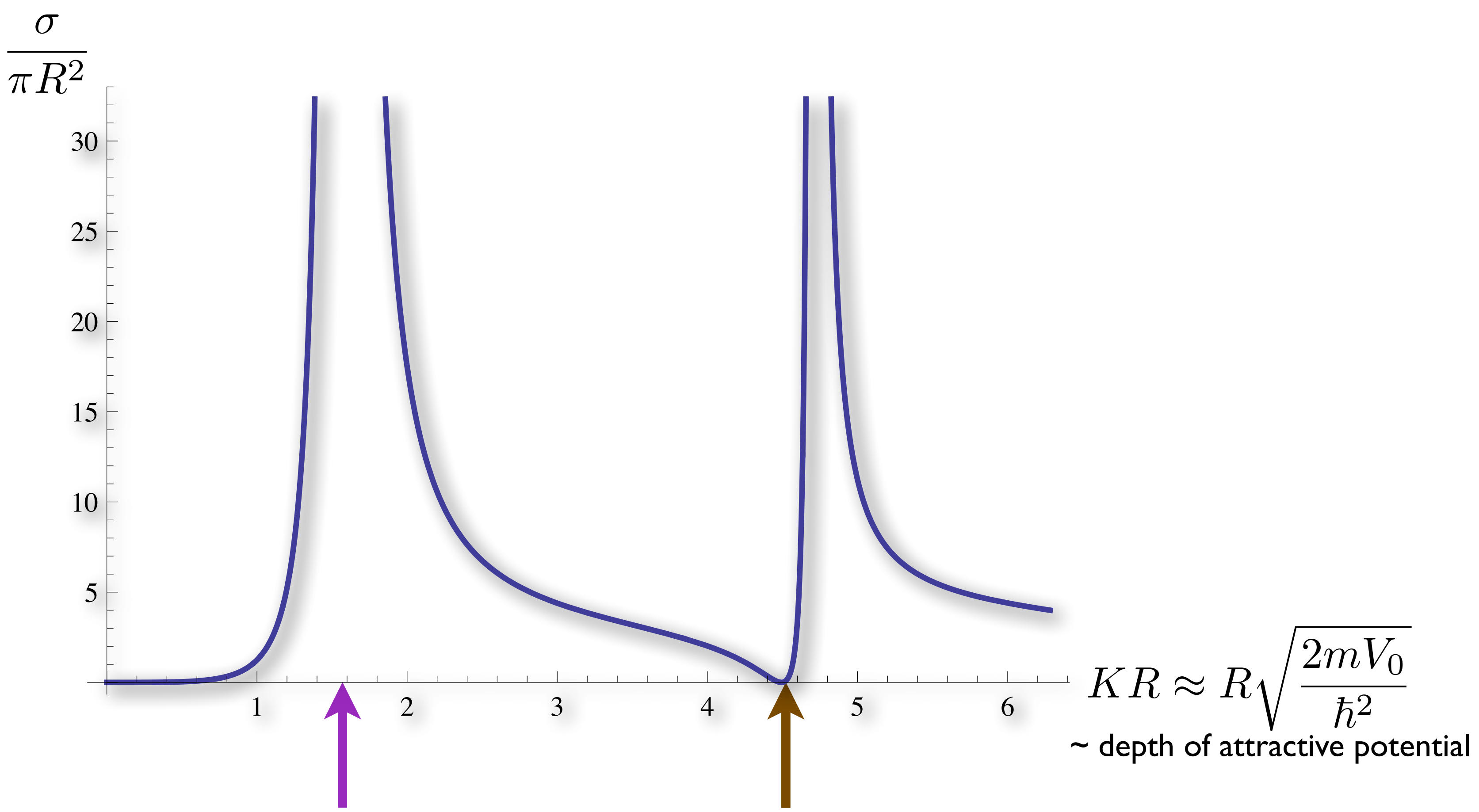
$$\sigma = 4\pi|f|^2 = 4\pi R^2 \left(\frac{\tan KR}{KR} - 1\right)^2$$

Scattering length

散乱長

$$a = -f(k = 0) \qquad\qquad \sigma(k \simeq 0) = 4\pi a^2$$

$$\sigma = 4\pi|f|^2 = 4\pi R^2\left(\frac{\tan KR}{KR} - 1\right)^2$$



$KR = \pi/2$
infinite scattering length

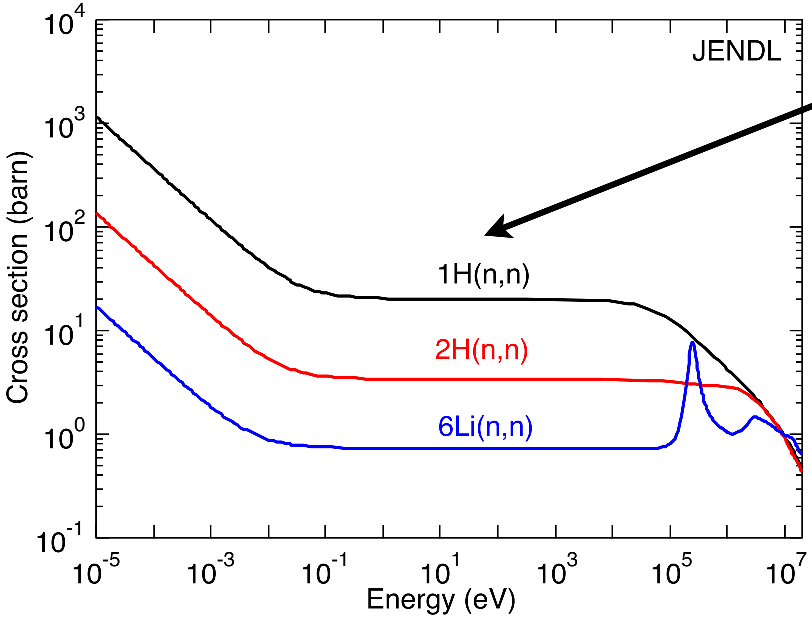
$\sigma \gg \pi R^2$

no scattering

“Ramsauer-Townsend effect”
ラムザウアー・タウンゼント効果

Nucleon-nucleon effect

核子-核子散乱

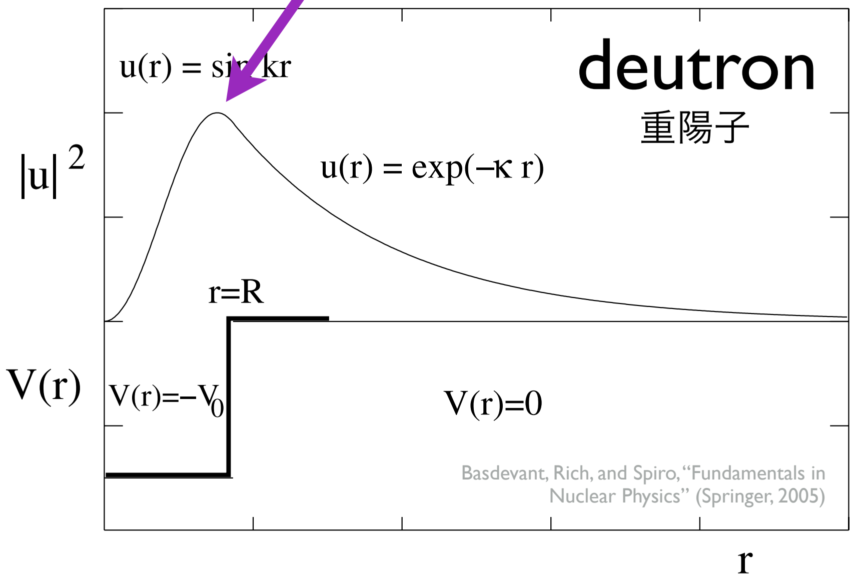


range of the strong interaction

flat region

$$\sigma_{el} \sim 20\,\mathrm{b} \gg \pi(2\mathrm{fm})^2 \sim 0.1\,\mathrm{b}$$

$kR \approx \pi/2$ ➡ V_0R^2~109 MeV fm^2



Basdevant, Rich, and Spiro, "Fundamentals in Nuclear Physics" (Springer, 2005)

Nucleon-nucleon effect

核子-核子散乱

	f (fm)	R (fm)	V_0 (MeV)	V_0R^2 (MeV fm^2)
n–p (s=1, T=0)	$+5.423 \pm 0.005$	1.73 ± 0.02	46.7	139.6
n–p (s=0, T=1)	-23.715 ± 0.015	2.73 ± 0.03	12.55	93.5
p–p (s=0, T=1)	-17.1 ± 0.2	2.794 ± 0.015	11.6	90.5
n–n (s=0, T=1)	-16.6 ± 0.6	2.84 ± 0.03	11.1	89.5

Basdevant, Rich, and Spiro, "Fundamentals in Nuclear Physics" (Springer, 2005)

$$\sigma_{n-p} = \frac{3}{4}4\pi|f_{s=1}|^2 + \frac{1}{4}4\pi|f_{s=0}|^2 \approx 20\,\mathrm{b}$$