Fundamentals in Nuclear Physics
原子核基礎

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Nuclear decays and fundamental interactions

Four fundamental interactions

interaction 相互作用	exchanged particle (gauge boson)	decay 壊変
gravity 重力	graviton 重力子	
weak 弱い相互作用	$W^{\pm}$, Z^0	beta decay
electromagnetic 電磁相互作用	photon 光子	gamma decay
strong 強い相互作用	gluon グルーオン	
nuclear force 核力	pion and other hadrons	

alpha decay ← tunnel effect

壊変（崩壊）速度　自然幅

Decay rate, natural width

probability to decay in an interval *dt*

$$dP = \frac{dt}{\tau} = \lambda dt$$

decay rate 壊変（崩壊）速度 (λ)

mean life time 平均寿命 (τ)

number of unstable nuclei $N(t) = N(t=0)e^{-t/\tau}$

half life 半減期 $t_{1/2} = (\ln 2)\tau = 0.693\tau$

$$^{7}\mathrm{Li}\,(7.459\,\mathrm{MeV}) \to \mathrm{n}\,^{6}\mathrm{Li},\ ^{3}\mathrm{H}\,^{4}\mathrm{He} \qquad \tau = 6\times 10^{-21}\ \mathrm{sec}$$

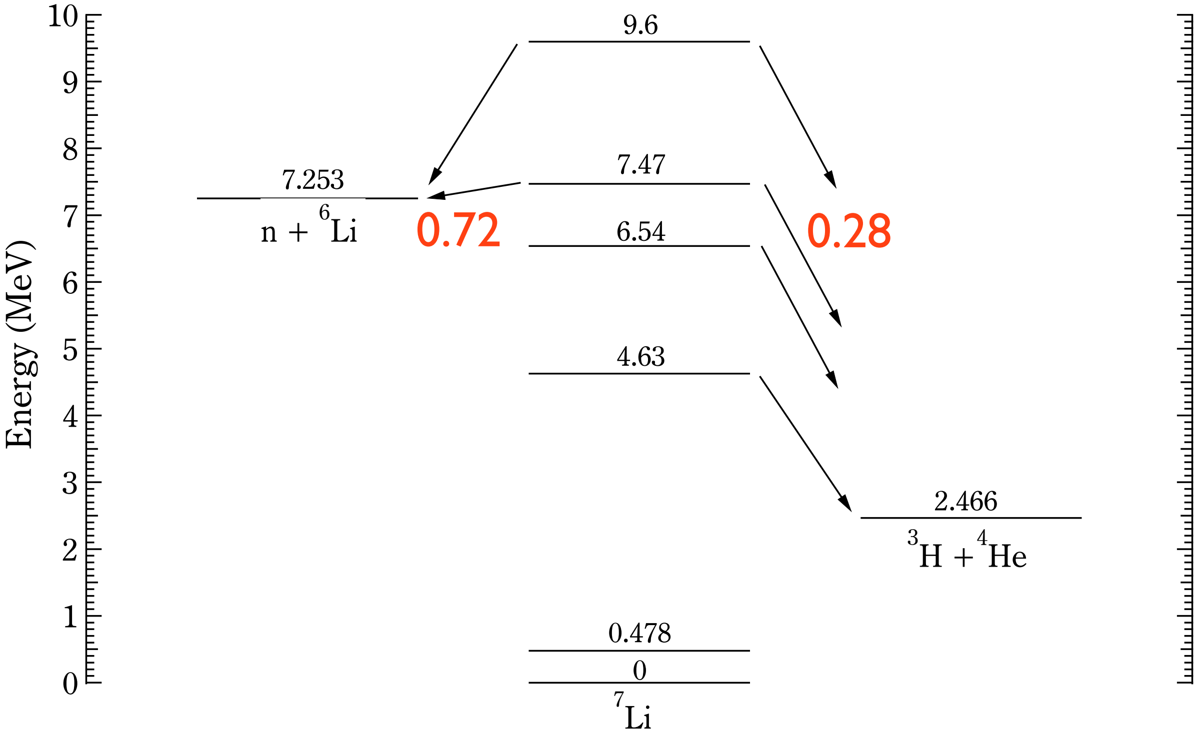
$$^{76}\mathrm{Ge} \to {}^{76}\mathrm{Se}\,2\mathrm{e}^{-}\,2\bar{\nu}_e \qquad t_{1/2} = 1.78\times 10^{21}\ \mathrm{yr}$$ > 10^{11} × (age of universe) !

An unstable particle has an energy uncertainty or "natural width"

$$\Gamma = \hbar\lambda = \frac{\hbar}{\tau} = \frac{6.58\times 10^{-22}\ \mathrm{MeV\ sec}}{\tau}$$

分岐比
Branching ratio

- Often, an unstable state (nucleus, isotope) has more than one decay channels.



channel k

branching ratio

$$B_k$$

$$\sum_k B_k = 1$$

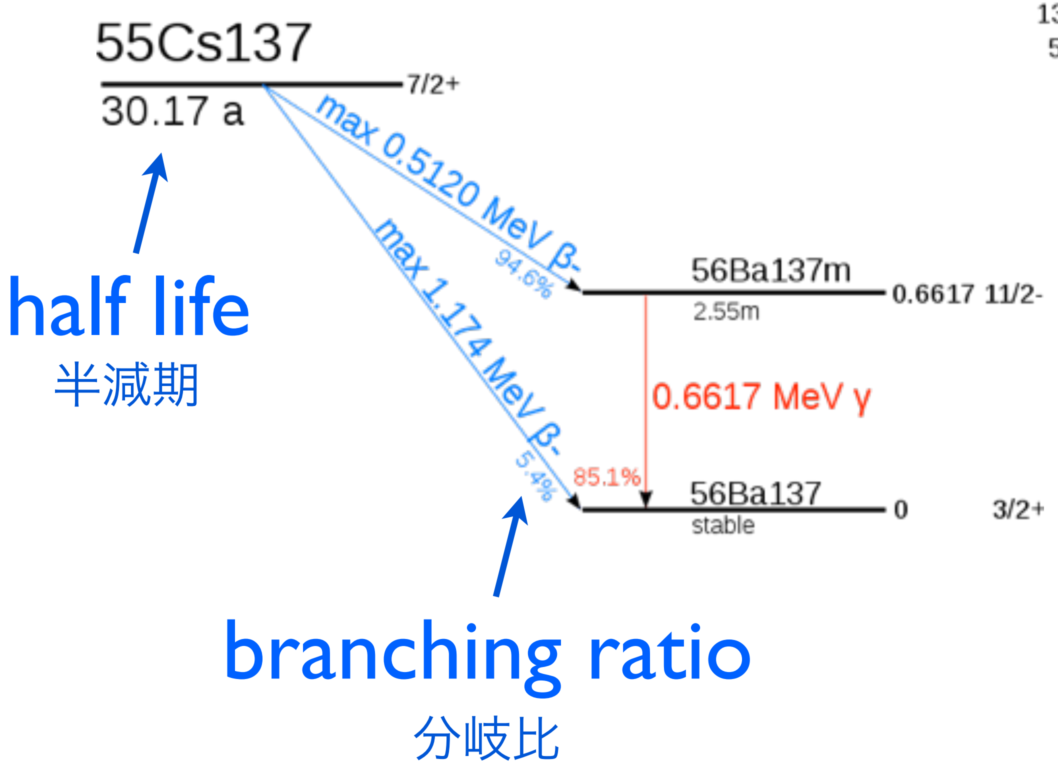
partial decay rate

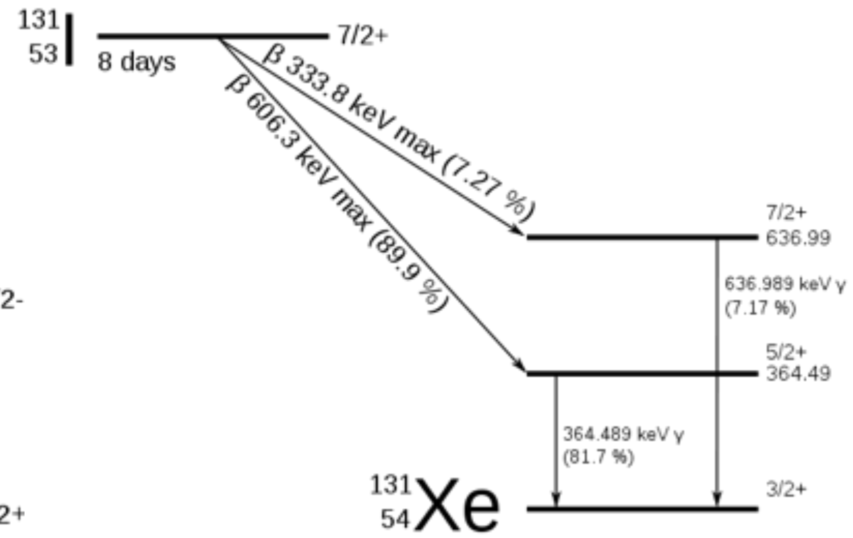
$$\lambda_k = B_k \lambda \qquad \sum_k \lambda_k = \lambda$$

partial width 部分幅

$$\Gamma_k = B_k \Gamma \qquad \sum_k \Gamma_k = \Gamma$$

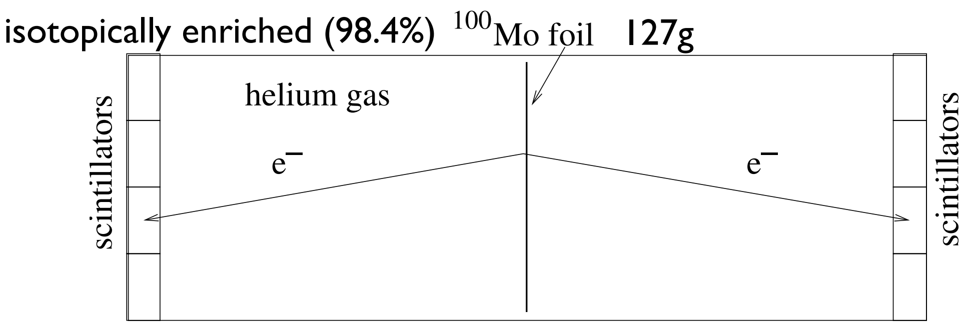
壊変図
Decay diagram



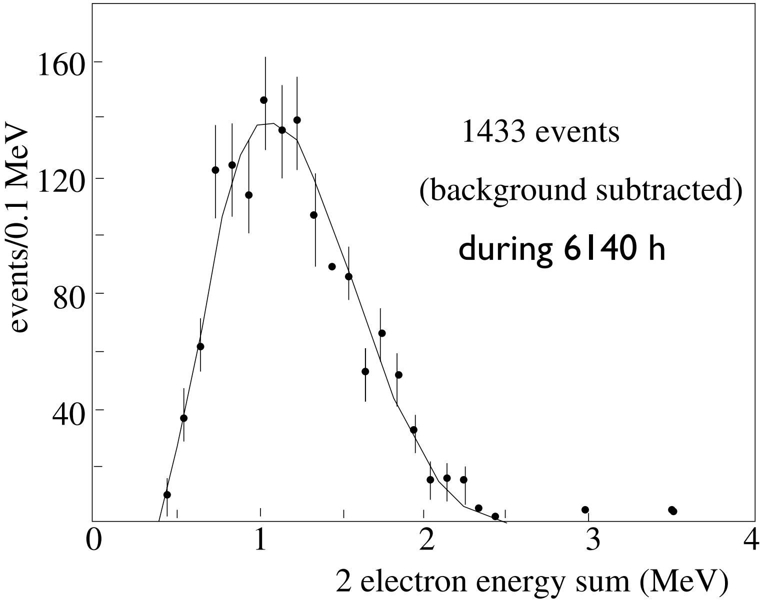


Measurement of half life 半減期の測定

$\tau > 10^8$ yr (α decay, double β decay)



Basdevant, Rich, and Spiro, "Fundamentals in Nuclear Physics" (Springer, 2005)



Basdevant, Rich, and Spiro, "Fundamentals in Nuclear Physics" (Springer, 2005)

- still present on Earth
- can be chemically and isotopically isolated in macroscopic quantity
- detected decays, quantity → lifetime

$^{100}\mathrm{Mo} \rightarrow {}^{100}\mathrm{Ru}\, 2e^- \; 2\bar{\nu}_e$ double β decay

half-life: $(0.95\pm0.11)\times10^{19}$ yr

10 min < τ < 10^8 yr (α decay, β decay)

- no longer present on Earth and must be produced in nuclear reactions
- purify chemically or isotopically
- detect decays and derive τ

10^{-10} s < τ < 10^3 s (α decay, β decay, γ decay)

- chemical and isotopic purification impossible
- particles produced in nuclear reactions, slowed down, and stopped
- detect decays and derive τ

τ < 10^{-10} s (γ decay, dissociation)

- standard timing techniques not applicable
- a variety of ingenious techniques: Doppler-shift attenuation method, Mössbauer spectroscopy

壊変速度の計算式

Formula for decay rates

decay $a \xrightarrow{T} \underbrace{b_1 + b_2 + \cdots + b_N}$

interaction 変化を引き起こす相互作用

静止質量 rest mass M
energy $E = Mc^2$

$|f\rangle$ state of final particles 終状態

decay rate probability per unit time that a decays into f
粒子 a が単位時間に状態 f に壊変する確率

$$\lambda_{a\to f} = \frac{2\pi}{\hbar}\left|\langle f|T|a\rangle\right|^2 \delta\left(Mc^2 - \sum_j E_j\right)$$

transition matrix element
遷移行列要素

energy conservation
エネルギー保存

Fermi's golden rule
フェルミの黄金則

Gamma decay

ガンマ壊変（崩壊）

Energetics

エネルギーについての考察

gamma decay

$$\mathrm{A}^* \rightarrow \mathrm{A} + \gamma$$

gamma ray　ガンマ線

spontaneous emission　自然放出

unstable high-energy state

(stable) low-energy state

$$m_{A*} > m_A \qquad m_{A*} - m_A \ll m_A$$

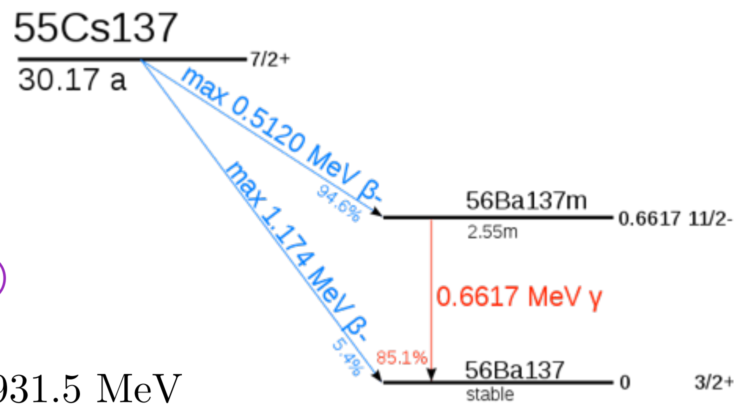
momentum conservation　$p = \dfrac{E_\gamma}{c}$　運動量保存

energy conservation　エネルギー保存

$$E_\gamma + \frac{p^2}{2m_A} = (m_{A*} - m_A)\, c^2$$

recoil energy (energy loss)
反跳エネルギー（エネルギー損失）

$$E_R = \frac{E_\gamma^2}{2m_A c^2} \qquad m_A c^2 \simeq A \times 931.5\ \mathrm{MeV}$$



$E_R \ll E_\gamma$　　$E_\gamma \simeq (m_{A*} - m_A)\, c^2$　　but　$E_R > \Gamma$　in general

Emitted gamma rays are not resonantly re-absorbed by other nuclei in gases

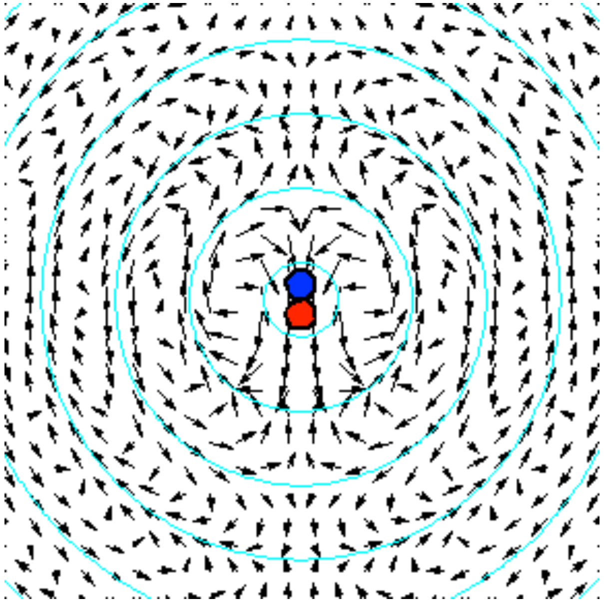
電気双極子遷移

Electric-dipole transitions

Classical image　古典電磁気学的なイメージ

radiation from an oscillating electric dipole

振動する電気双極子からの古典的な放射



http://www.eto.titech.ac.jp/contents/sub04/chapter02.html

Quantum mechanically　量子力学的には

rate $\lambda_{i\to f} = \frac{4\alpha}{3}\frac{q^2}{e^2}\frac{E_\gamma^3}{\hbar^3 c^2}|\langle f|\mathbf{r}|i\rangle|^2$

fine-structure constant 微細構造定数 $\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \simeq \frac{1}{137}$

$$\langle f|\mathbf{r}|i\rangle = \int d^3\mathbf{r}\,\psi_f^*(\mathbf{r})\mathbf{r}\psi_i(\mathbf{r})$$

Atomic transition

$$\hbar\omega \sim \mathrm{eV} \quad \langle r\rangle \sim 10^{-10}\,\mathrm{m} \quad \tau \sim 10^{-9} - 10^{-7}\,\mathrm{s} \quad \Gamma = \hbar/\tau \sim 10^{-7}\,\mathrm{eV} \ll \hbar\omega$$
$$\gg E_R = E_\gamma^2/(2m_A c^2) \sim 10^{-9}\,\mathrm{eV}$$

Nuclear transition $\langle r\rangle \sim A^{1/3}10^{-15}\,\mathrm{m}$ ➡ $\lambda(E1) \sim \frac{\alpha E_\gamma^3}{\hbar}\left(\frac{A^{1/3}\,\mathrm{fm}}{\hbar c}\right)^2$

$$E_\gamma \sim \mathrm{MeV} \quad \tau \sim 10^{-17} - 10^{-15}\,\mathrm{s} \quad \Gamma \sim 10\,\mathrm{eV} \ll E_\gamma$$

多重極遷移

Higher multi-pole transitions

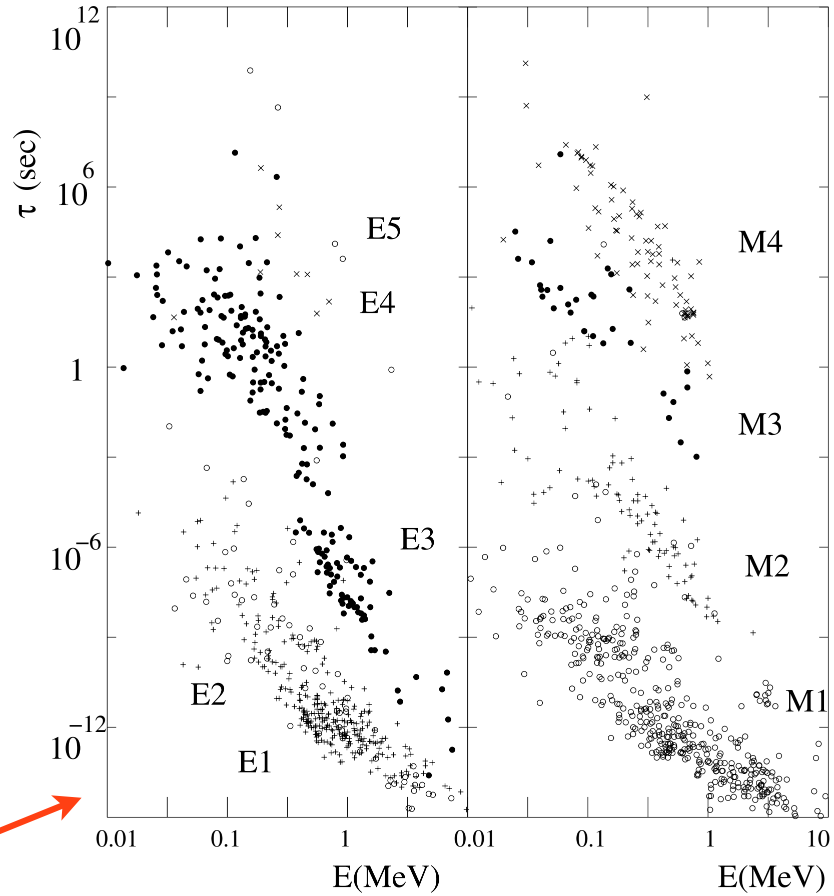
Often, electric-dipole (E1) decay is forbidden. $\langle f|\mathbf{r}|i\rangle = 0$

→ may still decay radiatively by higher-order and slower processes

Table 4.1. Selection rules for radiative transitions

type	symbol	angular momentum change $\lvert\Delta J\rvert \le$	parity change
electric dipole	E1	1	yes
magnetic dipole	M1	1	no
electric quadrupole	E2	2	no
magnetic quadrupole	M2	2	yes
electric octopole	E3	3	yes
magnetic octopole	M3	3	no
electric 16-pole	E4	4	no
magnetic 16-pole	M4	4	yes

Basdevant, Rich, and Spiro, "Fundamentals in Nuclear Physics" (Springer, 2005)



Lifetime of excited nuclear states as a function of E_γ for various multipoles

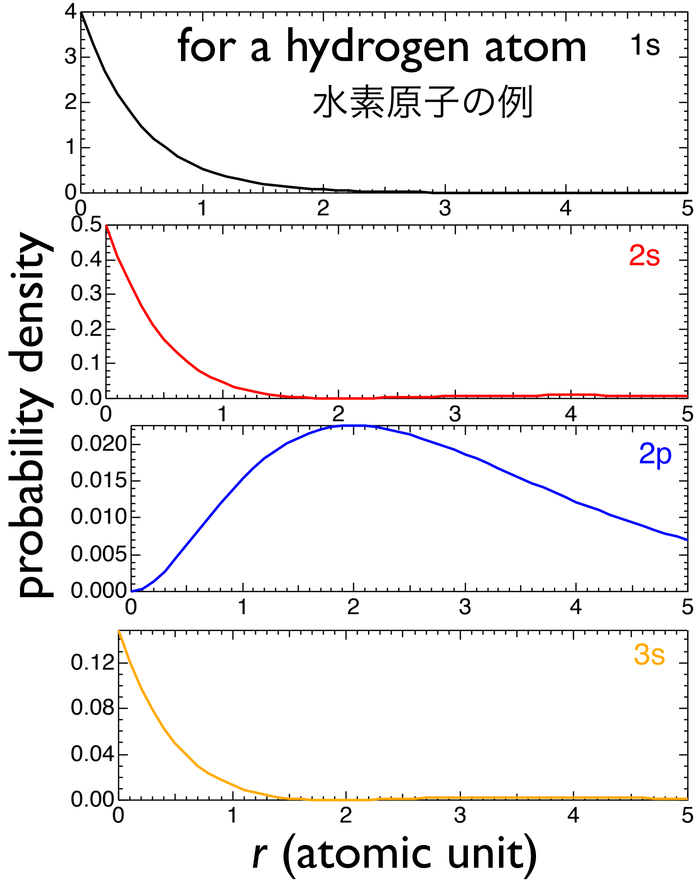
Basdevant, Rich, and Spiro, "Fundamentals in Nuclear Physics" (Springer, 2005)

内部転換
Internal conversion

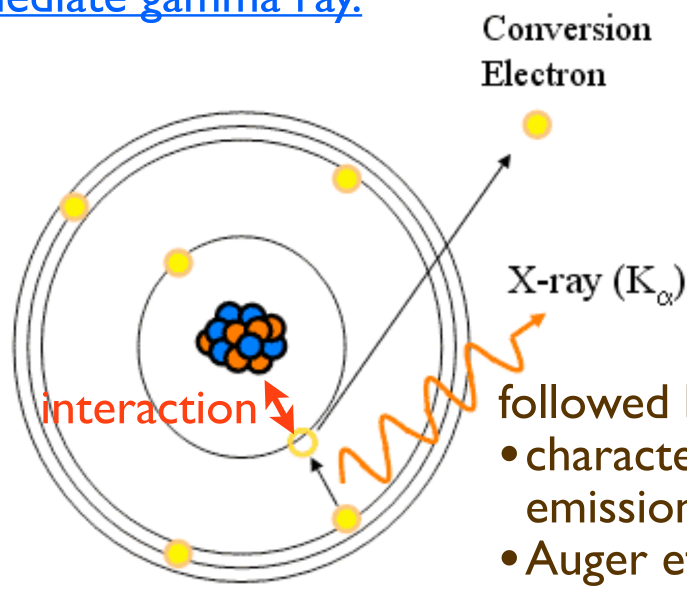
An excited nucleus can interact with an electron in one of the lower atomic orbitals, causing the electron to be emitted (ejected) from the atom.

s-electrons have finite probability density at the nuclear position.

s軌道の電子は、原子核の位置で存在確率が有限



The electron may couple to the excited state of the nucleus and take the energy of the nuclear transition <u>directly, without an intermediate gamma ray.</u>

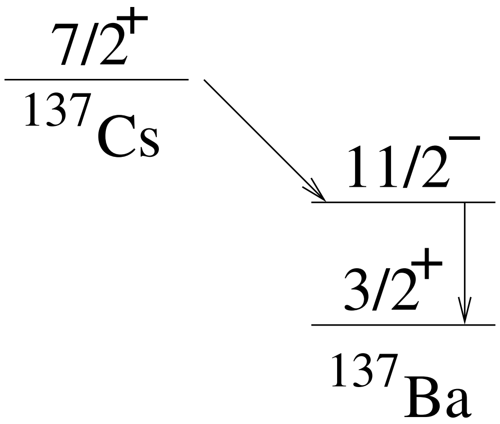


followed by
- characteristic x-ray emission 特性X線放出
- Auger effect オージェ効果

Energy of the conversion electron

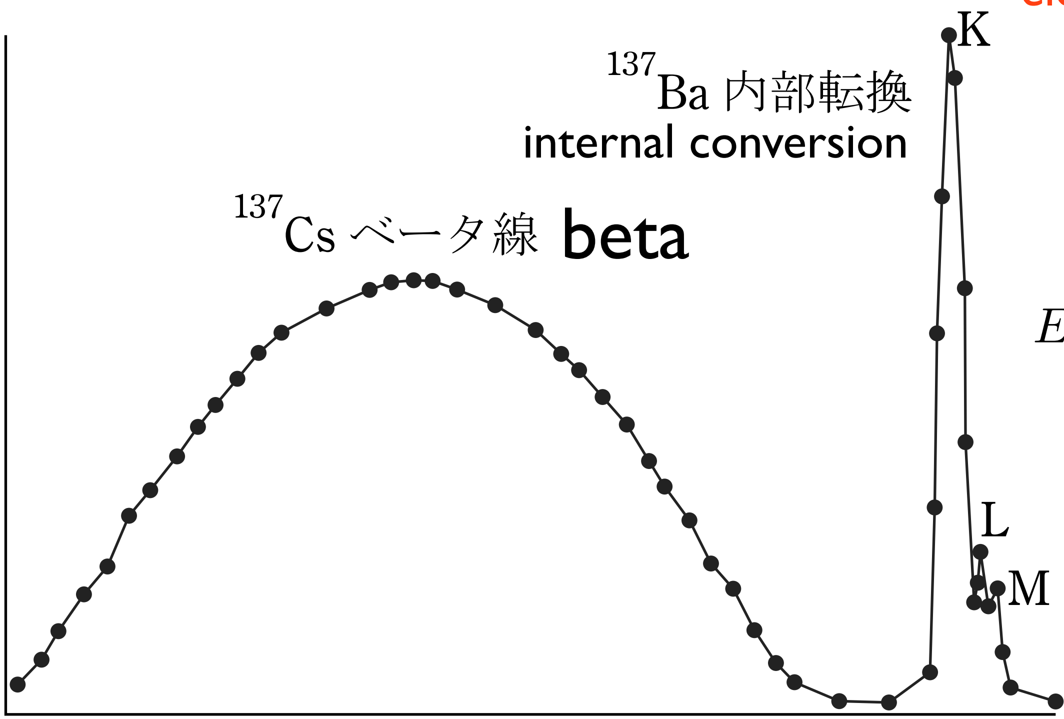
$$E_{ce} \simeq (m_{A*} - m_A)\,c^2 - E_b \simeq E_\gamma - E_b$$

binding energy of the electron



γ (90%)
internal conversion (10%)

most internal conversion electrons from the *K* shell
K殻からの内部転換電子が支配的



$$E_{ce} \simeq (m_{A*} - m_A)\, c^2 - E_b$$

ejection from higher orbitals generally less probable
L, M殻からの放出は一般的に少ない

電子運動量 electron momentum

メスバウアー効果

Mössbauer effect

recoil energy (energy loss)

反跳エネルギー（エネルギー損失）

$$E_R = \frac{E_\gamma^2}{2m_A c^2}$$

Emitted gamma rays are not resonantly re-absorbed by other nuclei in gases.

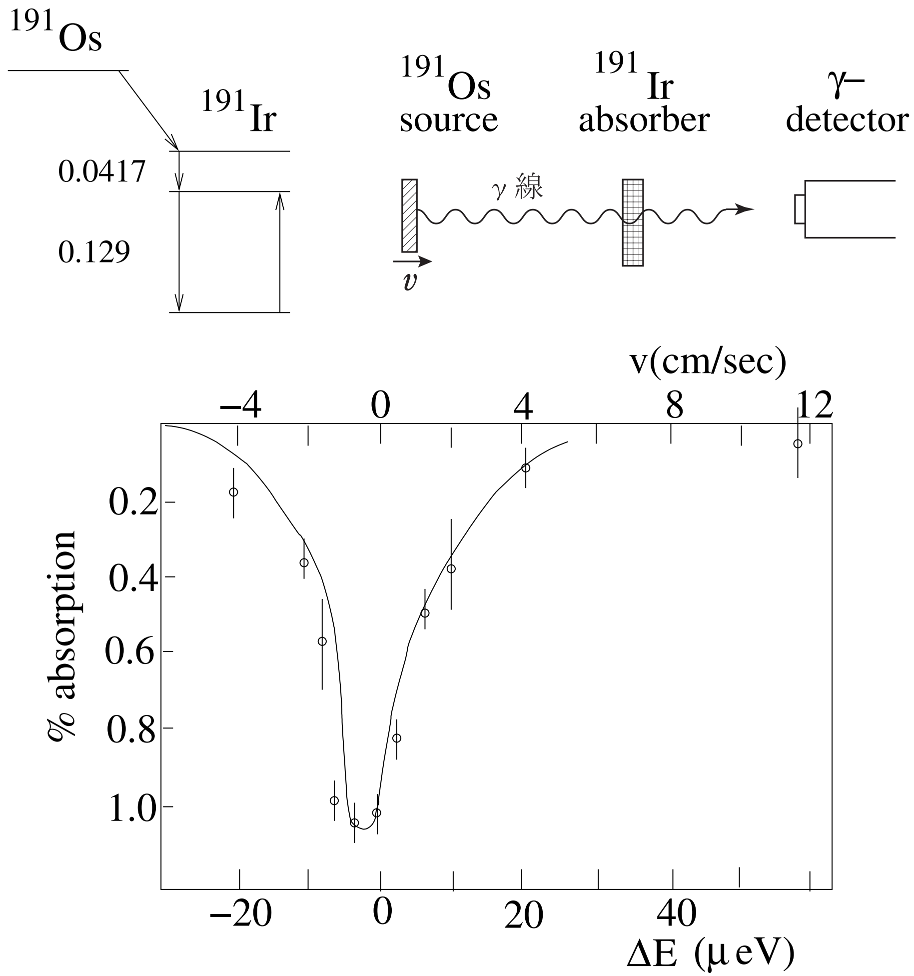
↓ but ...

Inverse transition (resonant re-absorption) possible when

- nuclear recoil is suppressed in a crystal ("very very large m_A") ← Mössbauer effect (discovered in 1957)
- the excited nucleus decays in flight with the Doppler effect compensating the nuclear recoil

メスバウアー分光による寿命測定

Mössbauer spectroscopy



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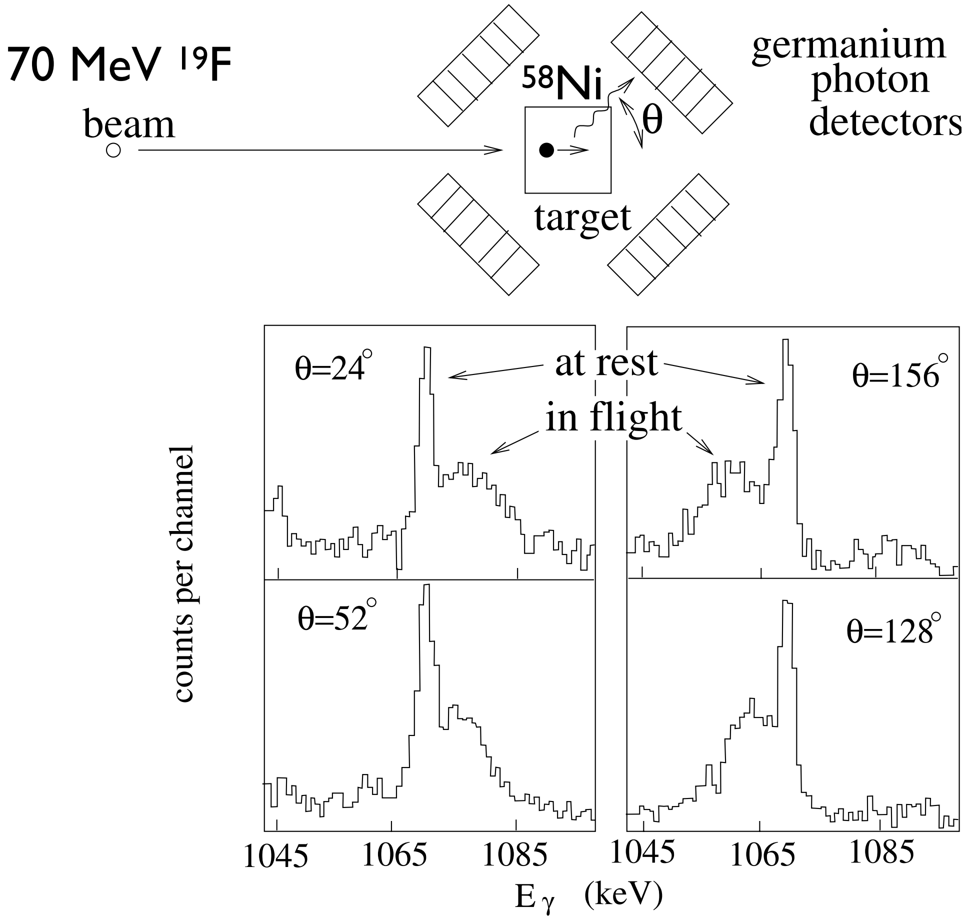
Doppler-shift attenuation method



Basdevant, Rich, and Spiro, "Fundamentals in Nuclear Physics" (Springer, 2005)

^{74}Br 1068 keV gamma-ray ➡ 0.25 ps lifetime

メスバウアー効果　　ドップラーシフト

Mössbauer effect + Doppler shift ➡

一般相対性理論の検証

Test of Albert Einstein's theory of general relativity

by Pound and Rebka, 1959

- Gravitational red shift of light
- Clocks run differently at different places in a gravitational field

Gravitational shift

$$h(f_r - f_e) = mgH$$

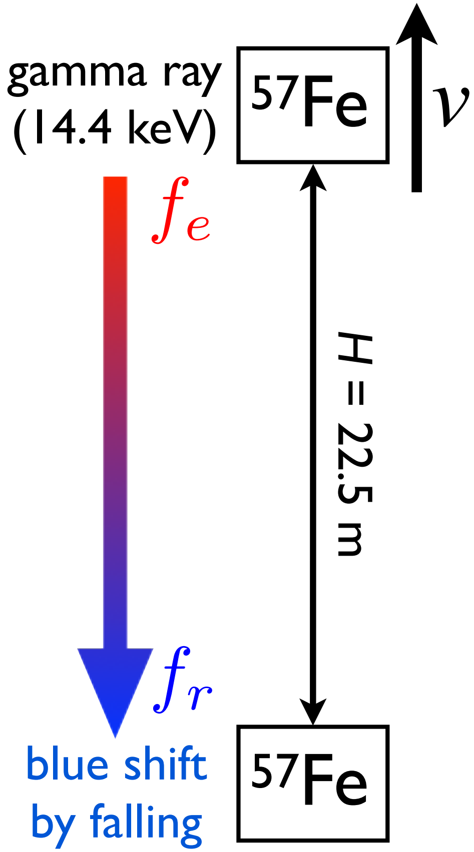
$$hf_e = mc^2$$

$$\frac{f_r}{f_e} = 1 + \frac{gH}{c^2}$$

Doppler shift

$$\frac{f_r}{f_e} = \sqrt{\frac{1 - v/c}{1 + v/c}} \approx 1 - \frac{v}{c}$$

$$\Rightarrow \quad v = \frac{gH}{c} = 7.36 \times 10^{-7}\,\mathrm{m/s}$$



Jefferson Laboratory
(Harvard University)

https://en.wikipedia.org/wiki/Pound%E2%80%93Rebka_experiment

Weak interaction and beta decay

弱い相互作用とベータ壊変（ベータ崩壊）

Four fundamental interactions

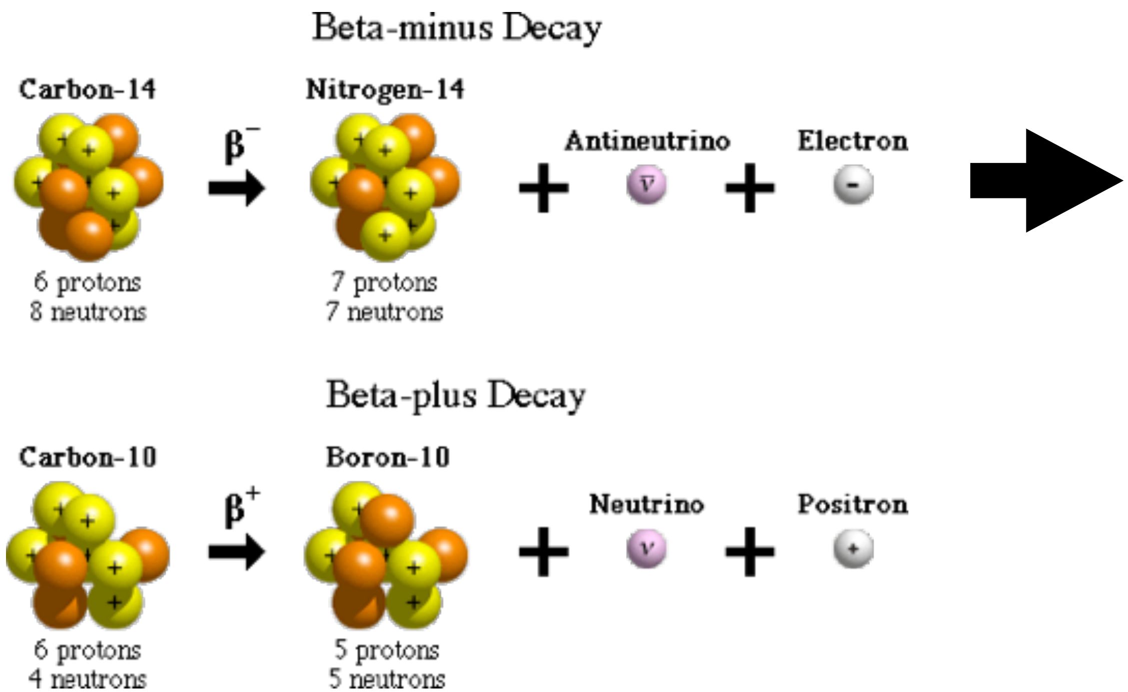
interaction 相互作用	exchanged particle (gauge boson)	decay 壊変
gravity 重力	graviton 重力子	
weak 弱い相互作用	$W^{\pm}$, Z^0	beta decay
electromagnetic 電磁相互作用	photon 光子	gamma decay
strong 強い相互作用	gluon グルーオン	
nuclear force 核力	pion and other hadrons	

alpha decay ← tunnel effect

beta decay

β^- decay $\quad {}^A_Z N \to {}^{A}_{Z+1}N' + \mathrm{e}^- + \bar{\nu}_e$

β^+ decay $\quad {}^A_Z N \to {}^{A}_{Z-1}N' + \mathrm{e}^+ + \nu_e$

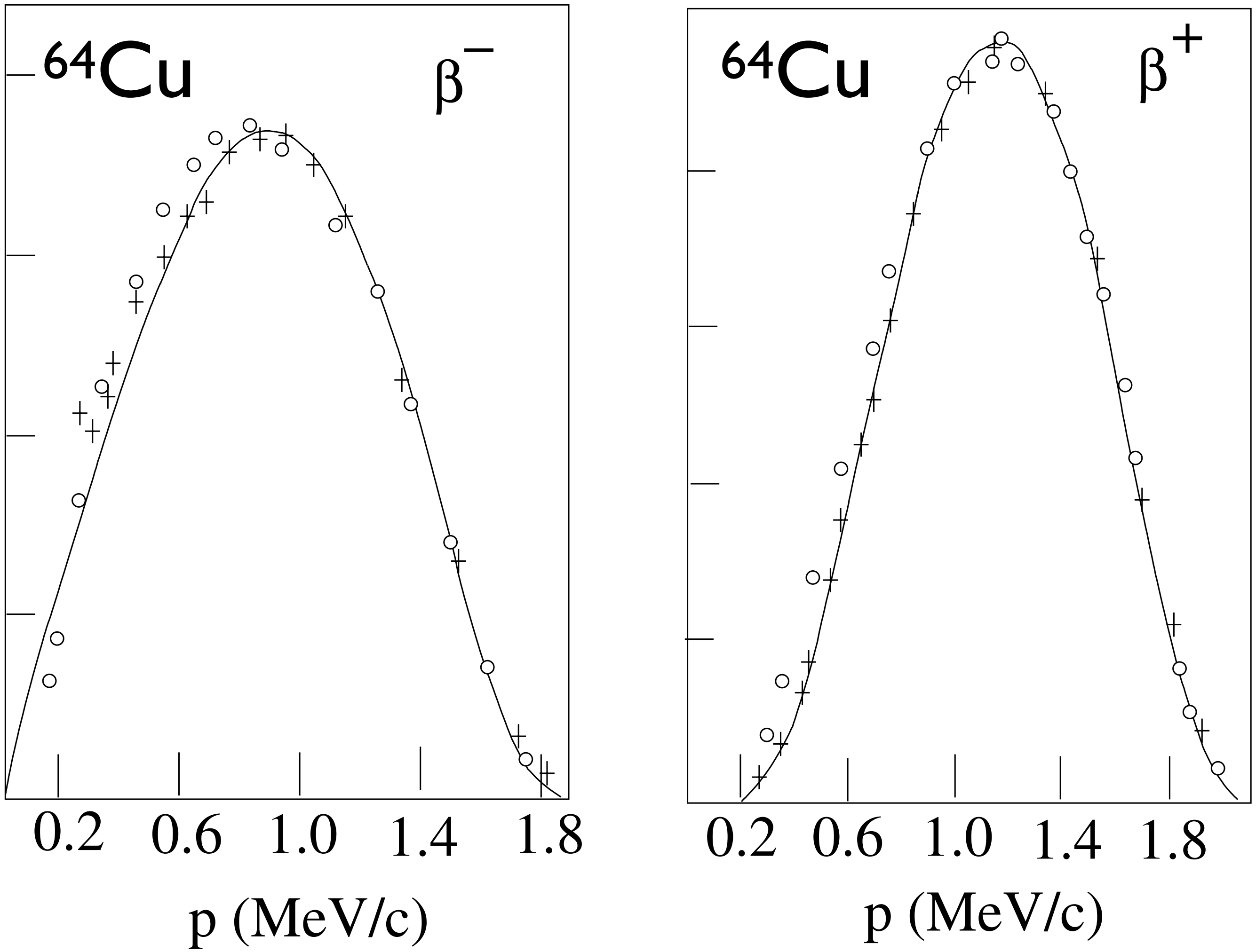


https://www.slideshare.net/yschhabra/radioactivity-45823825

half life = 5730 years

dating 年代測定

Emitted electron (positron) energy has a broad distribution



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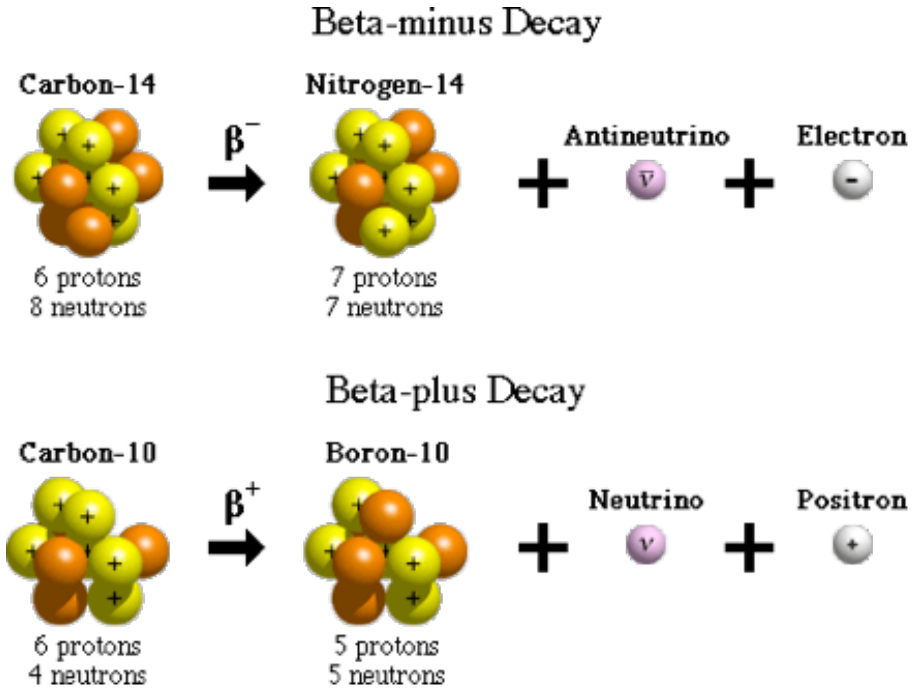
beta decay

$$\beta^- \ \text{decay} \qquad {}^A_Z N \to {}^{A}_{Z+1}N' + \mathrm{e}^- + \bar{\nu}_e$$

$$\beta^+ \ \text{decay} \qquad {}^A_Z N \to {}^{A}_{Z-1}N' + \mathrm{e}^+ + \nu_e$$



half life = 5730 years

dating 年代測定

https://upload.wikimedia.org/wikipedia/commons/4/43/Pauli.jpg

Pauli

The existence of the neutrino was predicted by Wolfgang Pauli in 1930 to explain how beta decay could conserve energy, momentum, and angular momentum.

fundamental processes

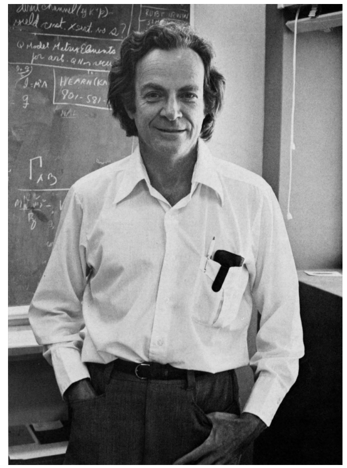
$$\mathrm{n} \longrightarrow \mathrm{p}\,\mathrm{e}^{-}\,\bar{\nu}_e$$

m_p = 938.3 MeV/c^2 < m_n = 939.6 MeV/c^2

mean life = 881.5 ± 1.5 s

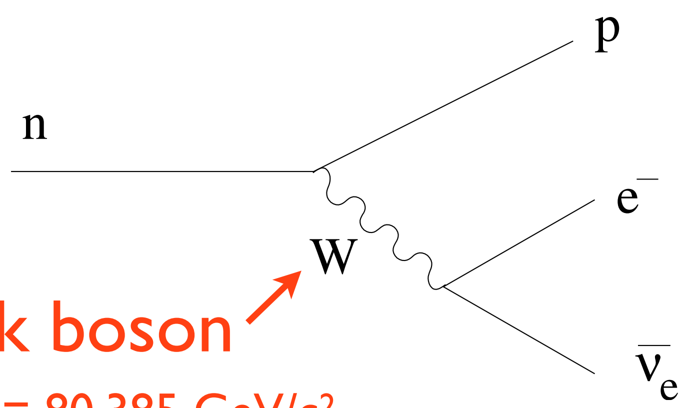
$$\mathrm{p} \longrightarrow \mathrm{n}\,\mathrm{e}^{+}\,\nu_e$$

- free proton does NOT decay
- takes place only in nuclei



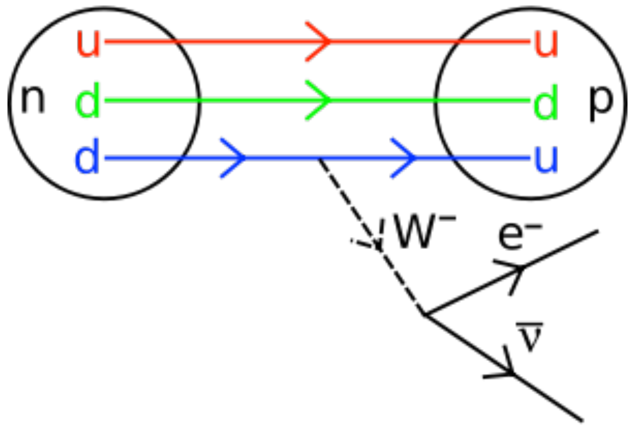
https://commons.wikimedia.org/wiki/File:Richard_Feynman_1988.png

Feynman diagram ファインマン図



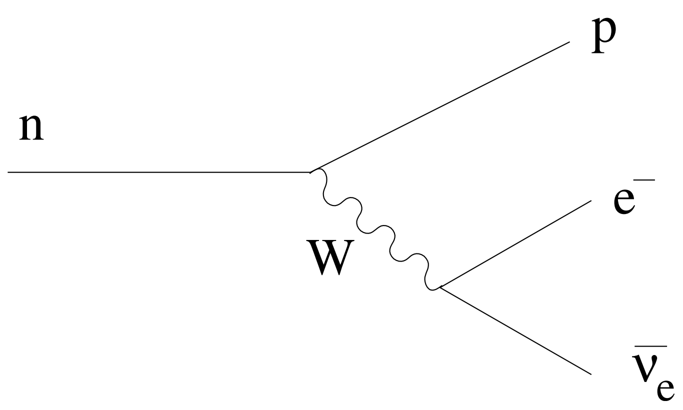
weak boson

m_W = 80.385 GeV/c^2



cf. m_{pion} = 139.570 MeV/c^2 (±), 134.9766 MeV/c^2 (neutral)

By transforming the Feynman diagram ...



$\mathrm{n} \longrightarrow \mathrm{p\,e^-}\,\bar{\nu}_e$	beta-
$\mathrm{p} \longrightarrow \mathrm{n\,e^+}\,\nu_e$	beta+
$\mathrm{p\,e^-} \longrightarrow \mathrm{n}\,\nu_e$	electron capture (EC)
$\bar{\nu}_e\,\mathrm{p} \longrightarrow \mathrm{e^+\,n}$	neutrino detection

Fermi theory of beta decay

Decay rate

$$w = \frac{2\pi}{\hbar} \left| \langle \psi_\mathrm{p} \psi_\mathrm{e} | H_\beta | \psi_\mathrm{n} \psi_\nu \rangle \right|^2 \frac{dn}{dE}$$

Fermi's golden rule

density of state 状態密度

$$\approx \int e^{-ik_\mathrm{p}\mathbf{r}_2} e^{-ik_\mathrm{e}\mathbf{r}_2} H_\beta\left(\mathbf{r}_2 - \mathbf{r}_1\right) e^{ik_\mathrm{n}\mathbf{r}_1} e^{ik_\nu \mathbf{r}_1} dV$$

weak interaction is a short-range force

$$H_\beta(\mathbf{r}_2 - \mathbf{r}_1) \sim G\delta(\mathbf{r}_2 - \mathbf{r}_1)$$

$$\approx G$$

Electron energy distribution dominated by density of state

放出される電子のエネルギー分布は状態密度で決まる

Density of state 状態密度

assuming plane waves

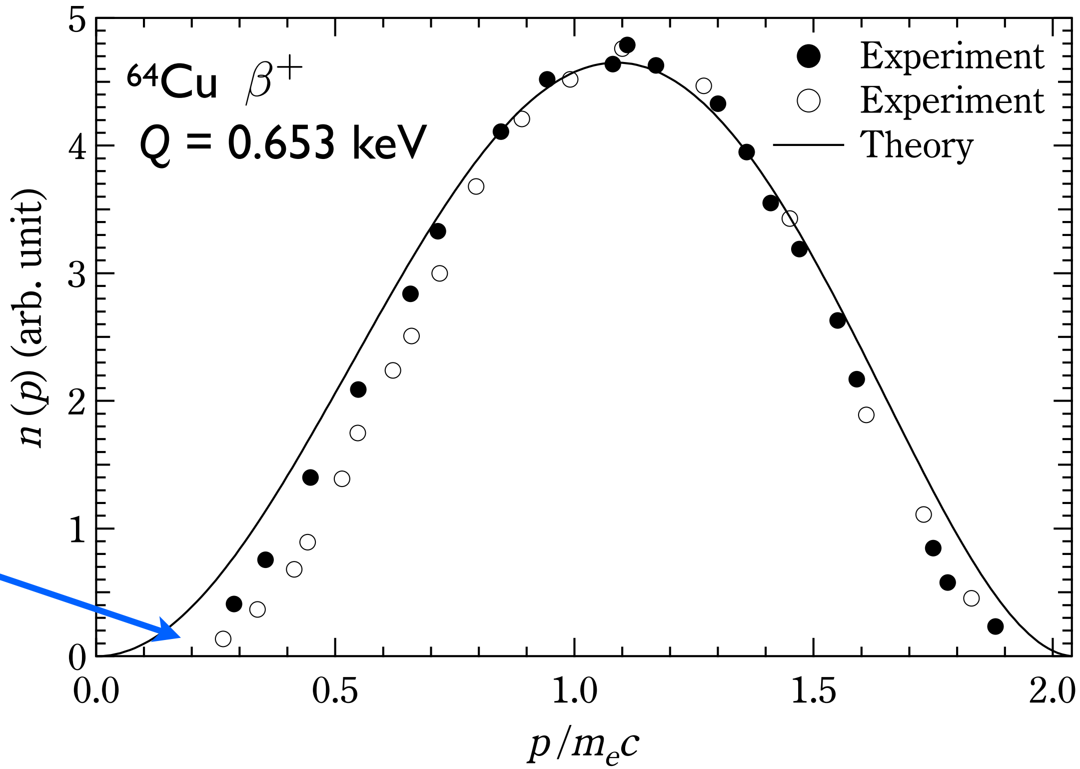
$$dn \propto p^2 dp q^2 dq$$

p : electron momentum
q : neutrino momentum

energy $Q = E_e + E_\nu$ (electron, neutrino) $\qquad E_\nu = cq \qquad E_e = \sqrt{m_e^2 c^4 + p^2 c^2}$

$$dE = dE_\nu = c dq \Rightarrow \frac{dn}{dE} \propto p^2 q^2 dp \propto (Q - E_e)^2 p^2 dp$$

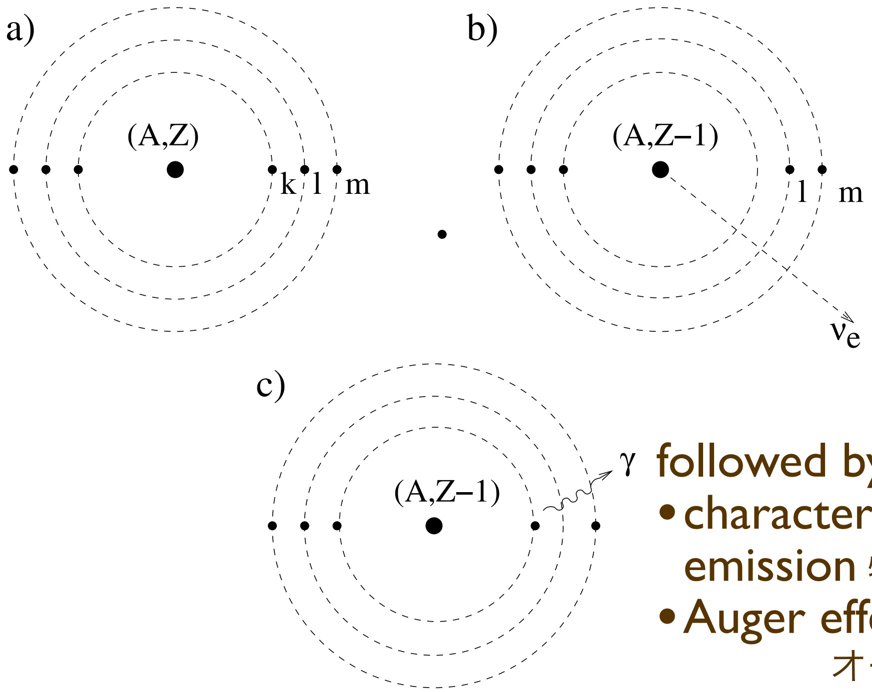
statistical factor 統計因子



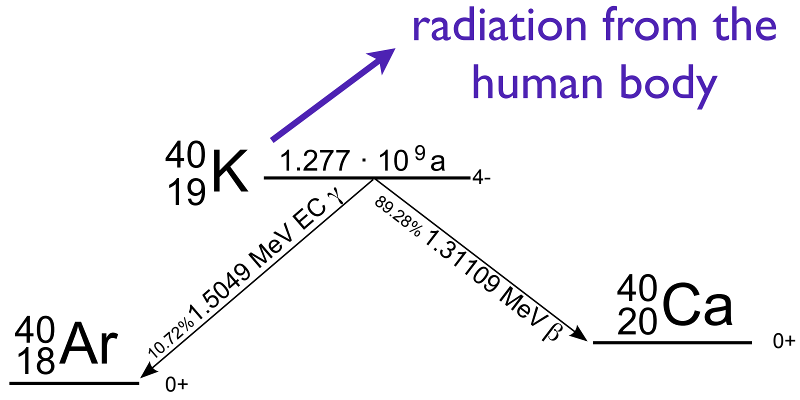
Coulomb repulsion

電子捕獲（軌道電子捕獲）

Electron capture (EC)



followed by
- characteristic x-ray emission 特性X線放出
- Auger effect オージェ効果



$$
{}^A_ZN + e^- \longrightarrow {}^{\;\;A}_{Z-1}N' + \nu_e
$$

fundamental process: $\mathrm{p\,e^-} \longrightarrow \mathrm{n}\,\nu_e$

neutrino energy: $E_\nu = M(A,Z)c^2 - M(A,Z-1)c^2$

atomic mass (*not* nuclear mass)

β⁺ decay and electron capture

β^+ decay $\quad {}^A_Z N \to {}^A_{Z-1}N' + \mathrm{e}^+ + \nu_e$

$$M_N(A,Z)c^2 > M_N(A,Z-1)c^2 + m_e c^2$$

nuclear mass

electron capture $\quad {}^A_Z N + e^- \to {}^A_{Z-1}N' + \nu_e$

$$M_N(A,Z)c^2 > M_N(A,Z-1)c^2 - m_e c^2$$

Both may not always be energetically possible!

Symmetry and conservation law

対称性と保存則

no change under a transformation

Any symmetry of a physical law has a corresponding conservation law

Noether's theorem ネーターの定理

symmetry	conserved quantity
temporal translation	energy
spatial translation 平行移動	momentum
rotation 回転	angular momentum
reflection **r**→-**r** (P) 空間反転	parity
time reversal (T) 時間反転	*T*-parity
charge conjugation (C) 粒子反粒子変換	*C*-parity
gauge invariance ゲージ不変性	electric charge

https://ja.wikipedia.org/wiki/エミー・ネーター

Example: Coulomb force $V(\mathbf{r}) = \dfrac{q_1 q_2}{4\pi\epsilon_0 |\mathbf{r}|^2}$ or $V(\mathbf{r}_1, \mathbf{r}_2) = \dfrac{q_1 q_2}{4\pi\epsilon_0 |\mathbf{r}_1 - \mathbf{r}_2|^2}$

example in the classical mechanics

Hamilton equations

$$\dot{q}_i = \frac{\partial H}{\partial p_i} \qquad \dot{p}_i = -\frac{\partial H}{\partial q_i}$$

If the Hamiltonian does not explicitly depend on q_i (invariant under the spatial translation)

$$\Rightarrow \quad \dot{p}_i = 0 \quad \Rightarrow \quad p_i = \mathrm{const}$$

Conservation of momentum 運動量保存

gauge invariance ゲージ不変性

$$\mathbf{B} = \nabla \times \mathbf{A}, \quad \mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \nabla\phi$$

invariant under the gauge transformation

$$\mathbf{A} \to \mathbf{A}' = \mathbf{A} + \nabla\chi, \quad \phi' = \phi - \frac{\partial \chi}{\partial t}$$

Invariance of the Action S 作用素積分

$\Rightarrow$ Conservation of the electric charge $\dfrac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = 0$

Parity

reflection

$$\hat{\pi}\psi(\mathbf{r}) = \psi(-\mathbf{r})$$

parity operator

$$\hat{\pi}^2\psi(\mathbf{r}) = \psi(\mathbf{r})$$

Eigenvalues → ± 1

If the physical law is invariant under the reflection (gravitational, electromagnetic, and strong interaction)

$$i\hbar\frac{\partial}{\partial t}\hat{\pi}\psi = H\hat{\pi}\psi \qquad i\hbar\frac{\partial}{\partial t}\hat{\pi}\psi = \hat{\pi}H\psi$$

$$\Rightarrow \quad \hat{\pi}H = H\hat{\pi} \quad \Rightarrow \quad [\hat{\pi}, H] = 0$$

Heisenberg's equation of motion

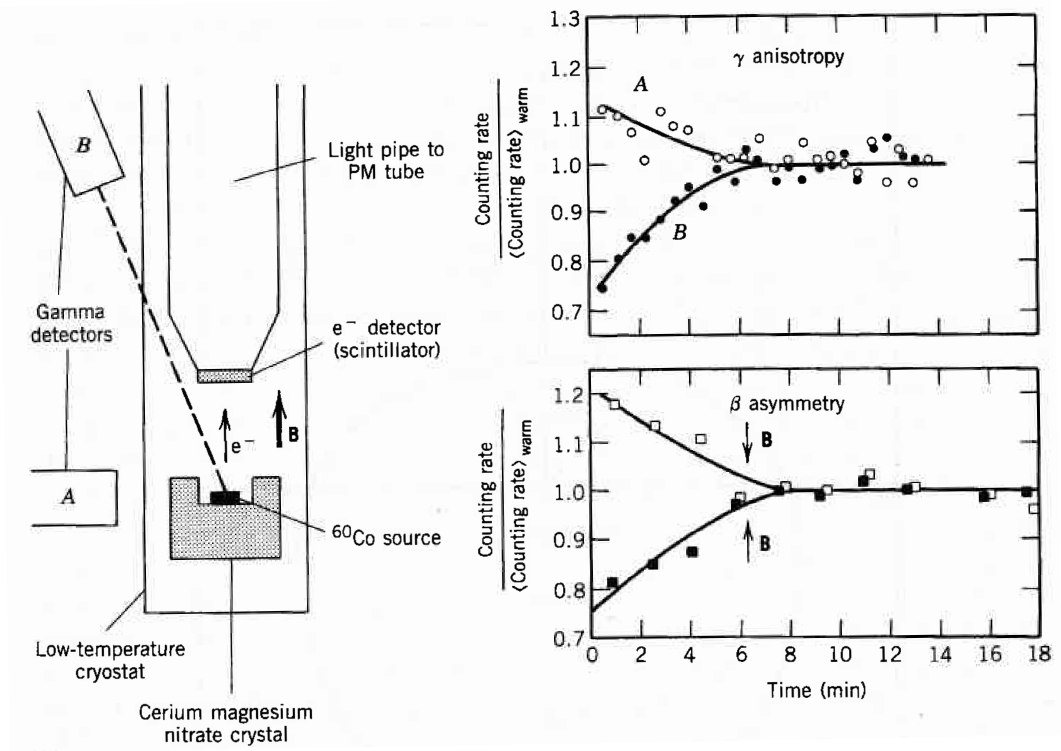
$$i\hbar\frac{d\hat{\pi}}{dt} = [\hat{\pi}, H] = 0$$

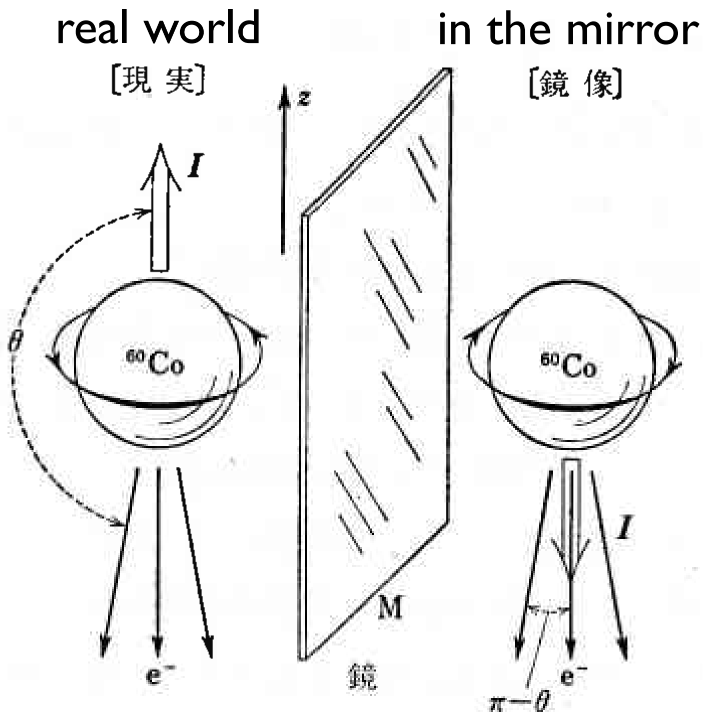
Conservation of parity

parity violation パリティ非保存
nonconservation of parity

in the weak interaction

- Prediction by T.-D. Lee and C. N. Yang in 1956
- Experimental verification by C.S. Wu in 1957





八木浩輔「原子核物理学」

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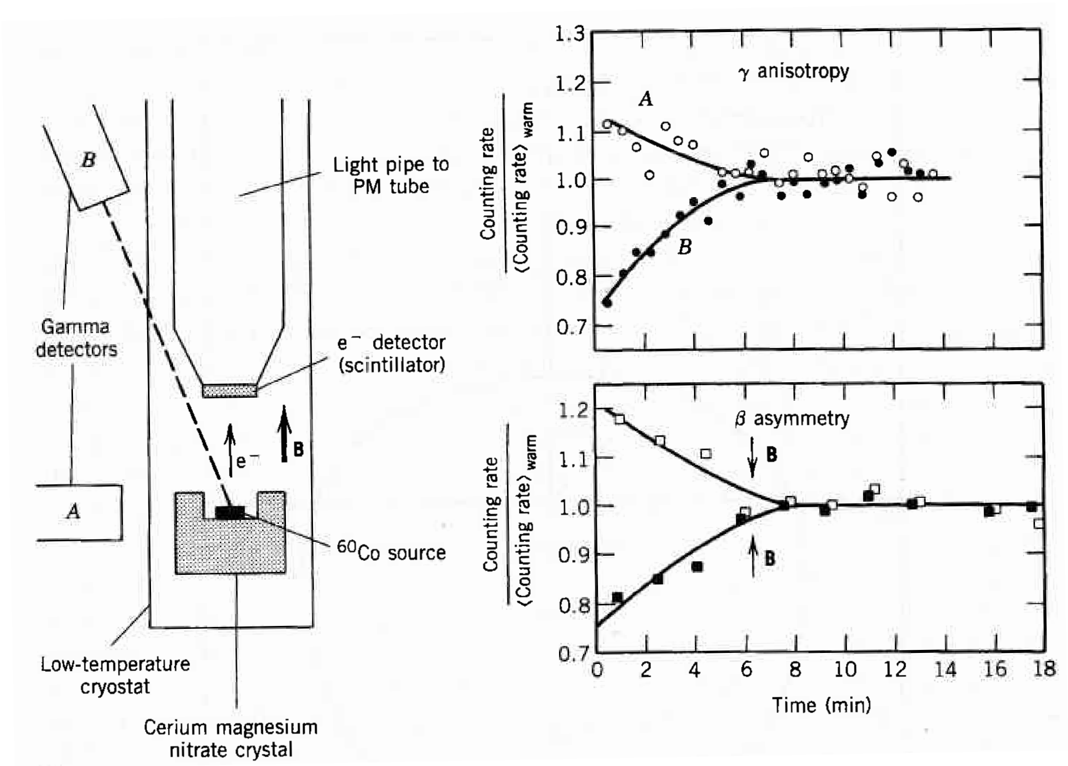

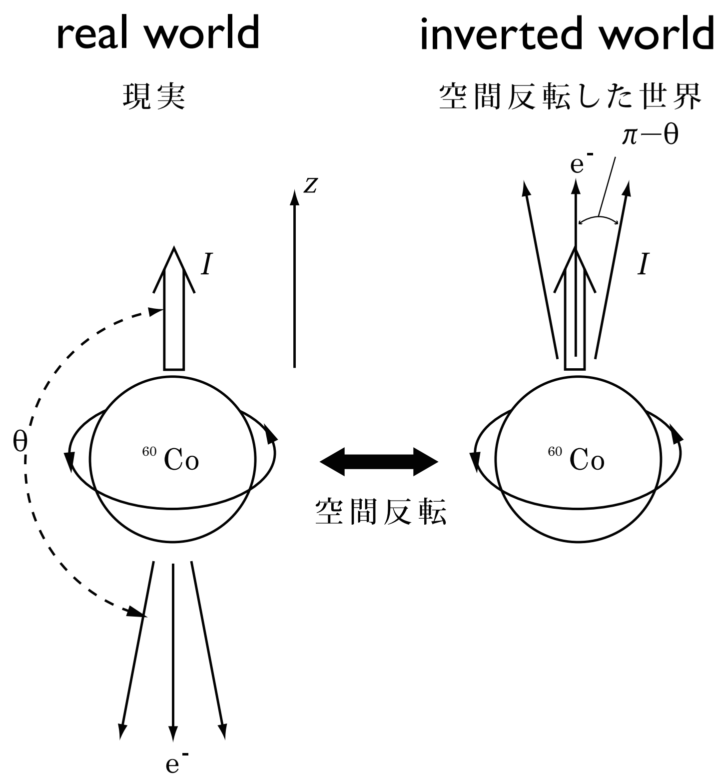

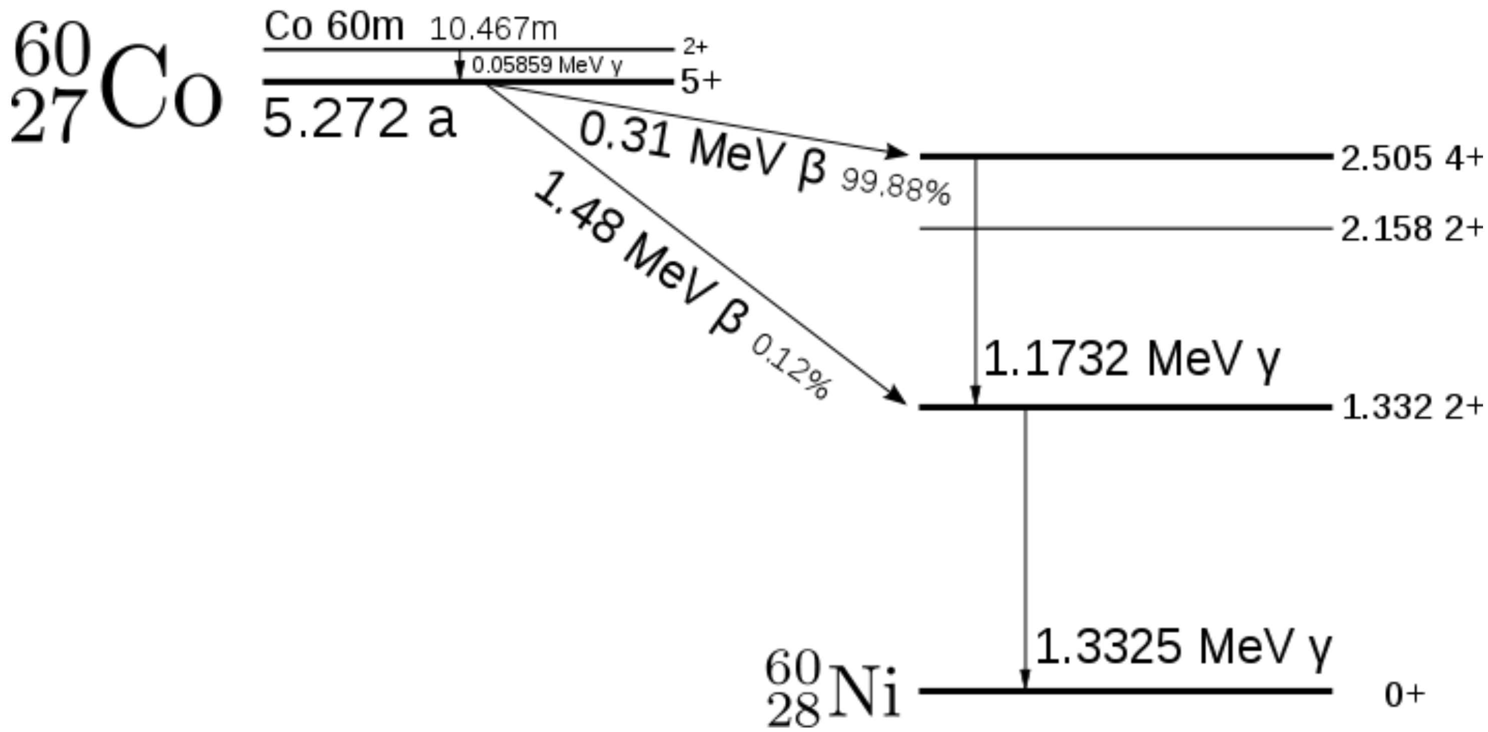
60 27 Co
Co 60m
10.467m
2+
0.05859 MeV γ
5+
5.272 a
0.31 MeV β 99.88%
1.48 MeV β 0.12%
2.505 4+
2.158 2+
1.1732 MeV γ
1.332 2+
1.3325 MeV γ
60 28 Ni
0+

https://en.wikipedia.org/wiki/Tsung-Dao_Lee

Lee

https://en.wikipedia.org/wiki/Yang_Chen-Ning

Yang

https://en.wikipedia.org/wiki/Chien-Shiung_Wu

Wu

Nobel prize in physics (1957)

CP violation

https://en.wikipedia.org/wiki/Makoto_Kobayashi_(physicist)

Makoto Kobayashi

https://en.wikipedia.org/wiki/Toshihide_Maskawa

Toshihide Maskawa

Nobel prize in physics (2008)

CPT theorem

CPT定理

- Preservation of CPT symmetry by all physical phenomena
- Any Lorentz invariant local quantum field theory with a Hermitian Hamiltonian must have CPT symmetry