Nuclear decays and fundamental interactions
## Four fundamental interactions

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Decay rate, natural width

probability to decay in an interval $dt$

$$dP = \frac{dt}{\tau} = \lambda dt$$

decay rate 壓變（崩壊）速度

mean life time 平均壽命

number of unstable nuclei

$$N(t) = N(t = 0)e^{-t/\tau}$$

half life 半減期

$$t_{1/2} = (\ln 2)\tau = 0.693\tau$$

$^7\text{Li} (7.459 \text{ MeV}) \rightarrow n\,^6\text{Li}, \quad ^3\text{H}\,^4\text{He} \quad \tau = 6 \times 10^{-21} \text{ sec}$

$^{76}\text{Ge} \rightarrow ^{76}\text{Se} 2e^- 2\bar{\nu}_e \quad t_{1/2} = 1.78 \times 10^{21} \text{ yr} \quad > 10^{11} \times \text{(age of universe)}$

An unstable particle has an energy uncertainty or "natural width"

$$\Gamma = \hbar \lambda = \frac{\hbar}{\tau} = \frac{6.58 \times 10^{-22} \text{ MeV sec}}{\tau}$$
Branching ratio

- Often, an unstable state (nucleus, isotope) has more than one decay channels.

\[
\begin{align*}
\lambda_k &= B_k \lambda \\
\sum_k \lambda_k &= \lambda \\
\Gamma_k &= B_k \Gamma \\
\sum_k \Gamma_k &= \Gamma
\end{align*}
\]
Decay diagram

55Cs137

half life

branching ratio

55Cs137
30.17 a

max 0.5120 MeV \( \beta^- \) max 1.114 MeV \( \beta^- \) 94.6%

56Ba137m
2.59 m

0.6617 MeV \( \gamma \)

85.1%

56Ba137
stable

131
53
8 days

7/2+

131
54

Xe

7/2+

364.489 keV \( \gamma \) (81.7%)

5/2+

354.49

636.99

7/2+

3/2+

636.989 keV \( \gamma \) (71.7%)

5/2+

3/2+

0.6617 MeV \( \gamma \)

3/2+

7/2+

8 days
Measurement of half life

$\tau > 10^8$ yr (α decay, double β decay)

- still present on Earth
- can be chemically and isotopically isolated in macroscopic quantity
- detected decays, quantity $\rightarrow$ lifetime

$^{100}$Mo $\rightarrow$ $^{100}$Ru $2e^- 2\bar{\nu}_e$  

half-life: $(0.95\pm 0.11) \times 10^{19}$ yr
10 min < $\tau$ < $10^8$ yr ($\alpha$ decay, $\beta$ decay)

- no longer present on Earth and must be produced in nuclear reactions
- purify chemically or isotopically
- detect decays and derive $\tau$

$10^{-10}$ s < $\tau$ < $10^3$ s ($\alpha$ decay, $\beta$ decay, $\gamma$ decay)

- chemical and isotopic purification impossible
- particles produced in nuclear reactions, slowed down, and stopped
- detect decays and derive $\tau$

$\tau$ < $10^{-10}$ s ($\gamma$ decay, dissociation)

- standard timing techniques not applicable
- a variety of ingenious techniques: Doppler-shift attenuation method, Mössbauer spectroscopy
Formula for decay rates

\[ \lambda_{a\rightarrow f} = \frac{2\pi}{\hbar} |\langle f|T|a\rangle|^2 \delta \left( Mc^2 - \sum_j E_j \right) \]

- decay \[ a \rightarrow b_1 + b_2 + \cdots + b_N \]
- interaction 变化を引き起こす相互作用
- rest mass \( M \)
- energy \( E = Mc^2 \)
- \( |f\rangle \) 終状態
- state of final particles
- decay rate 粒子 \( a \) が単位時間に状態 \( f \) に変化する確率

Fermi’s golden rule フェルミの黄金則
Gamma decay
ガンマ壊変（崩壊）
Energetics

**Gamma Decay**

\[ A^* \rightarrow A + \gamma \]

- Unstable high-energy state
- (Stable) low-energy state

### Gamma Ray

\[ m_{A^*} > m_A \quad m_{A^*} - m_A \ll m_A \]

**Momentum Conservation**

\[ p = \frac{E_\gamma}{c} \]

**Energy Conservation**

\[ E_\gamma + \frac{p^2}{2m_A} = (m_{A^*} - m_A) c^2 \]

- Recoil energy (energy loss)

\[ E_R = \frac{E_\gamma^2}{2m_A c^2} \quad m_A c^2 \simeq A \times 931.5 \text{ MeV} \]

- Emitted gamma rays are not resonantly re-absorbed by other nuclei in gases
Electric-dipole transitions

Classical image  
radiation from an oscillating electric dipole

Quantum mechanically  
rate  \( \lambda_{i\rightarrow f} = \frac{4\alpha q^2 E_\gamma^3}{3 e^2 \hbar^3 c^2} |\langle f | r | i \rangle|^2 \)

fine-structure constant  
\( \alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c} \sim \frac{1}{137} \)

\( |\langle f | r | i \rangle| = \int d^3r \, \psi_f^*(r) r \psi_i(r) \)

Atomic transition  
\( \hbar\omega \sim \text{eV} \quad |r| \sim 10^{-10} \text{m} \quad \tau \sim 10^{-9} - 10^{-7} \text{s} \quad \Gamma = \hbar/\tau \sim 10^{-7} \text{eV} \ll \hbar\omega \)

\( \gg E_R = E_\gamma^2/(2m_A c^2) \sim 10^{-9} \text{eV} \)

Nuclear transition  
\( |r| \sim A^{1/3} 10^{-15} \text{m} \quad \lambda(E1) \sim \frac{\alpha E_\gamma^3}{\hbar} \left( \frac{A^{1/3} \text{fm}}{\hbar c} \right)^2 \)

\( E_\gamma \sim \text{MeV} \quad \tau \sim 10^{-17} - 10^{-15} \text{s} \quad \Gamma \sim 10 \text{eV} \ll E_\gamma \)
Higher multi-pole transitions

Often, electric-dipole (E1) decay is forbidden. $\langle f | r | i \rangle = 0$

\[ \text{may still decay radiatively by higher-order and slower processes} \]

**Table 4.1. Selection rules for radiative transitions**

| type                  | symbol | angular momentum change $|\Delta J| \leq$ | parity change |
|-----------------------|--------|-----------------------------------------|---------------|
| electric dipole       | E1     | 1                                       | yes           |
| magnetic dipole       | M1     | 1                                       | no            |
| electric quadrupole   | E2     | 2                                       | no            |
| magnetic quadrupole   | M2     | 2                                       | yes           |
| electric octopole     | E3     | 3                                       | yes           |
| magnetic octopole     | M3     | 3                                       | no            |
| electric 16-pole      | E4     | 4                                       | no            |
| magnetic 16-pole      | M4     | 4                                       | yes           |

**Figure 4.5.** Lifetime of excited nuclear states as a function of $E_\gamma$ for various multipoles.

Lifetime of excited nuclear states as a function of $E_\gamma$ for various multipoles
Internal conversion

An excited nucleus can interact with an electron in one of the lower atomic orbitals, causing the electron to be emitted (ejected) from the atom.

$s$-electrons have finite probability density at the nuclear position.

The electron may couple to the excited state of the nucleus and take the energy of the nuclear transition directly, without an intermediate gamma ray.

\[ E_{ce} \simeq (m_{A*} - m_A) c^2 - E_b \simeq E_\gamma - E_b \]

**Energy of the conversion electron**

binding energy of the electron

\[ r \text{ (atomic unit)} \]
4.3 Weak interactions

Whereas the electromagnetic interactions responsible for the radiative decays conserve the number of protons and the number of neutrons, the weak interactions transform protons to neutrons or vice versa as well as the numbers of charged leptons and numbers of neutrinos. The archetype of a weak decay is nuclear $\beta^-$-decay $(A, Z) \rightarrow (A, Z \pm 1) \text{e}^- (\text{e}^+ \bar{\nu}_e)$. $(4.63)$

Fermi gave a remarkably efficient theory of this process as soon as 1933. The structure of this theory became more profound in 1968 with the advent of
Mössbauer effect

recoil energy (energy loss) \[ E_R = \frac{E_\gamma^2}{2m_Ac^2} \]

Emitted gamma rays are not resonantly re-absorbed by other nuclei in gases.

but ...

Inverse transition (resonant re-absorption) possible when
• nuclear recoil is suppressed in a crystal ("very very large \(m_A\)") \(\leftarrow\) Mössbauer effect (discovered in 1957)
• the excited nucleus decays in flight with the Doppler effect compensating the nuclear recoil
Mössbauer spectroscopy

Fig. 4.4. Measurement of the width of the first excited state of $^{191}\text{Ir}$ through Mössbauer spectroscopy [39]. The excited state is produced by the $\beta^-$-decay of $^{191}\text{Os}$.

De-excitation photons can be absorbed by the inverse transition in a $^{191}\text{Ir}$ absorber. This resonant absorption can be prevented by moving the absorber with respect to the source with velocity $v$ so that the photons are Doppler shifted out of the resonance. Scanning in energy then amounts to scanning in velocity with $\Delta E_\gamma/E_\gamma = v/c$.

It should be noted that photons from the decay of free $^{191}\text{Ir}$ have insufficient energy to excite $^{191}\text{Ir}$ because nuclear recoil takes some of the energy (4.42). Resonant absorption is possible with $v = 0$ only if the $^{191}\text{Ir}$ nuclei is “locked” at a crystal lattice site so the crystal as a whole recoils. The nuclear kinetic energy $p^2/2m$ in (4.42) is modified by replacing the mass of the nucleus with the mass of the crystal. The photon then takes all the energy and has sufficient energy to excite the original state. This “Mössbauer effect” is not present for photons with $E > 200$ keV because nuclear recoil is sufficient to excite phonon modes in the crystal which take some of the energy and momentum.
Doppler-shift attenuation method

Fig. 4.3. Measurement of radiative-decay lifetimes by the "Doppler-shift attenuation method" [38]. The top figure is a simplified version of the apparatus used to measure the lifetimes of excited states of $^{74}$Br. A beam of 70 MeV $^{19}$F ionsimpinges upon a $^{58}$Ni target, producing a variety of nuclei in a variety of excited states. The target is sufficiently thick that the produced nuclei stop in the target. Depending on the lifetime of the produced excited state, the state may decay before stopping ("in-flight" decays) or at rest. The target is surrounded by germanium-diode detectors (the Euroball array) that measure the energy of the photons. The bottom figure shows the energy distribution of photons corresponding to the 1068 keV line of $^{74}$Br for four germanium diodes at different angles with respect to the beam direction. Each distribution has two components, a narrow peak corresponding to decays at rest and a broad tail corresponding to Doppler-shifted in-flight decays. Note that decays with $\theta > 90$ deg ($\theta > 90$ deg) have Doppler shifts that are positive (negative). Roughly half the decays are in-flight and half at-rest. Knowledge of the time necessary to stop a Br ion in the target allowed one to deduce a lifetime of 0.25 ps for the state that decays by emission of the 1068 keV gamma (Exercise 4.4).

$^{70}$MeV $^{19}$F beam
$^{58}$Ni target
$^{74}$Br 1068 keV gamma-ray
0.25 ps lifetime
Test of Albert Einstein's theory of general relativity by Pound and Rebka, 1959

- Gravitational red shift of light
- Clocks run differently at different places in a gravitational field

**Gravitational shift**

\[ h(f_r - f_e) = mgH \]
\[ hf_e = mc^2 \]
\[ \frac{f_r}{f_e} = 1 + \frac{gH}{c^2} \]

**Doppler shift**

\[ \frac{f_r}{f_e} = \sqrt{\frac{1 - v/c}{1 + v/c}} \approx 1 - \frac{v}{c} \]

\[ v = \frac{gH}{c} = 7.36 \times 10^{-7} \text{ m/s} \]
Weak interaction and beta decay
弱い相互作用とベータ壊変（ベータ崩壊）
## Four fundamental interactions

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*Alpha decay* → *tunnel effect*
beta decay

$\beta^- \text{ decay} \quad \frac{A}{Z}N \rightarrow \frac{A}{Z+1}N' + e^- + \bar{\nu}_e$

$\beta^+ \text{ decay} \quad \frac{A}{Z}N \rightarrow \frac{A}{Z-1}N' + e^+ + \nu_e$

Beta-minus Decay

Carbon-14 $\rightarrow$ Beta$^-$

$6$ protons
$8$ neutrons

Nitrogen-14 $\rightarrow$ Antineutrino + Electron

$7$ protons
$7$ neutrons

half life = 5730 years
dating 年代測定

Beta-plus Decay

Carbon-10 $\rightarrow$ Beta$^+$

$6$ protons
$4$ neutrons

Boron-10 $\rightarrow$ Neutrino + Positron

$5$ protons
$5$ neutrons
Emitted electron (positron) energy has a broad distribution
The existence of the neutrino was predicted by Wolfgang Pauli in 1930 to explain how beta decay could conserve energy, momentum, and angular momentum.
fundamental processes

\[ n \rightarrow p e^- \bar{\nu}_e \quad m_p = 938.3 \text{ MeV}/c^2 < m_n = 939.6 \text{ MeV}/c^2 \]

mean life = 881.5 ± 1.5 s

\[ p \rightarrow n e^+ \nu_e \quad - \text{free proton does NOT decay} \]

- takes place only in nuclei

**Feynman diagram**

weak boson

\[ m_W = 80.385 \text{ GeV}/c^2 \]

cf. \( m_{\text{pion}} = 139.570 \text{ MeV}/c^2 \) (±), 134.9766 MeV/c² (neutral)
By transforming the Feynman diagram ...

\[ n \rightarrow p e^- \bar{\nu}_e \quad \text{beta-} \]

\[ p \rightarrow n e^+ \nu_e \quad \text{beta+} \]

\[ p e^- \rightarrow n \nu_e \quad \text{electron capture (EC)} \]

\[ \bar{\nu}_e p \rightarrow e^+ n \quad \text{neutrino detection} \]
Fermi theory of beta decay

**Decay rate**

\[
\omega = \frac{2\pi}{\hbar} \left| \langle \psi_p \psi_e | H_\beta | \psi_n \psi_\nu \rangle \right|^2 \frac{dn}{dE}
\]

- **Fermi’s golden rule**
- **density of state**
- **weak interaction is a short-range force**
  \[
  H_\beta (\mathbf{r}_2 - \mathbf{r}_1) \sim G \delta(\mathbf{r}_2 - \mathbf{r}_1)
  \]

\[
\approx \int e^{-ik_p \mathbf{r}_2} e^{-ik_e \mathbf{r}_2} H_\beta (\mathbf{r}_2 - \mathbf{r}_1) e^{ik_n \mathbf{r}_1} e^{ik_\nu \mathbf{r}_1} dV
\]

Electron energy distribution dominated by density of state

放出される電子のエネルギー分布は状態密度で決まる
Density of state 状態密度

assuming plane waves

\[ dn \propto p^2 dp q^2 dq \]

\[ p : \text{electron momentum} \]
\[ q : \text{neutrino momentum} \]

energy

\[ Q = E_e + E_\nu \]

\[ E_\nu = cq \]
\[ E_e = \sqrt{m_e^2 c^4 + p^2 c^2} \]

d\[E = dE_\nu = cdq \]

\[ \frac{dn}{dE} \propto p^2 q^2 dp \propto (Q - E_e)^2 p^2 dp \]

statistical factor 統計因子

\[ ^{64}\text{Cu} \beta^+ \]
\[ Q = 0.653 \text{ keV} \]

Coulomb repulsion
Electron capture (EC)

Followed by
- characteristic x-ray emission  特性X線放出
- Auger effect オージェ効果

\[
\begin{align*}
\frac{A}{Z} N + e^- & \rightarrow \frac{A}{Z-1} N' + \nu_e \\
\text{fundamental process:} & \quad p e^- \rightarrow n \nu_e \\
\text{neutrino energy:} & \quad E_{\nu} = M(A, Z)c^2 - M(A, Z-1)c^2
\end{align*}
\]

atomic mass (not nuclear mass)
\[ \beta^+ \text{ decay} \quad \frac{A}{Z} N \rightarrow \frac{A}{Z-1} N' + e^+ + \nu_e \]

\[ M_N(A, Z)c^2 > M_N(A, Z - 1)c^2 + m_e c^2 \]

Both may not always be energetically possible!
Symmetry and conservation law

対称性と保存則
Any \textbf{symmetry} of a physical law has a corresponding \textbf{conservation law}.

\textbf{Noether’s theorem} (ネーターの定理)

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<tr>
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<tr>
<td>rotation</td>
<td>angular momentum</td>
</tr>
<tr>
<td>reflection $\mathbf{r} \rightarrow -\mathbf{r}$ (P)</td>
<td>parity</td>
</tr>
<tr>
<td>time reversal (T)</td>
<td>$T$-parity</td>
</tr>
<tr>
<td>charge conjugation (C)</td>
<td>$C$-parity</td>
</tr>
<tr>
<td>gauge invariance</td>
<td>electric charge</td>
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Example: Coulomb force \[ V(\mathbf{r}) = \frac{q_1 q_2}{4 \pi \epsilon_0 |\mathbf{r}|^2} \quad \text{or} \quad V(\mathbf{r}_1, \mathbf{r}_2) = \frac{q_1 q_2}{4 \pi \epsilon_0 |\mathbf{r}_1 - \mathbf{r}_2|^2} \]
example in the classical mechanics

Hamilton equations

\[ \dot{q}_i = \frac{\partial H}{\partial p_i}, \quad \dot{p}_i = -\frac{\partial H}{\partial q_i} \]

If the Hamiltonian does not explicitly depend on \( q_i \) (invariant under the spatial translation)

\[ \dot{p}_i = 0 \quad \Rightarrow \quad p_i = \text{const} \]

Conservation of momentum

\[ \text{gauge invariance} \]

\[ B = \nabla \times A, \quad E = -\frac{\partial A}{\partial t} - \nabla \phi \]

invariant under the gauge transformation

\[ A \rightarrow A' = A + \nabla \chi, \quad \phi' = \phi - \frac{\partial \chi}{\partial t} \]

Invariance of the Action

\[ \text{Conservation of the electric charge} \]

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot j = 0 \]
Parity

If the physical law is invariant under the reflection (gravitational, electromagnetic, and strong interaction)

\[ \hat{\pi} \psi(\mathbf{r}) = \psi(-\mathbf{r}) \]

Eigenvalues \( \to \pm 1 \)

\[ \hat{\pi}^2 \psi(\mathbf{r}) = \psi(\mathbf{r}) \]

Heisenberg’s equation of motion

\[ i\hbar \frac{\partial}{\partial t} \hat{\pi} \psi = H \hat{\pi} \psi \]

\[ i\hbar \frac{\partial}{\partial t} \hat{\pi} \psi = \hat{\pi} H \psi \]

\[ \hat{\pi} H = H \hat{\pi} \]

\[ [\hat{\pi}, H] = 0 \]

Conservation of parity
parity violation 
nonconservation of parity

in the weak interaction

• Prediction by T.-D. Lee and C. N. Yang in 1956
• Experimental verification by C. S. Wu in 1957
parity violation  パリティ非保存

nonconservation of parity  一の弾性不全

in the weak interaction

• Prediction by T.-D. Lee and C. N. Yang in 1956
• Experimental verification by C.S. Wu in 1957

Before we leave this topic, we should discuss the effect of the p nonconservation on nuclear spectroscopy. The interaction between nucleons in a nucleus consists of two parts: the "strong" part, which arises primarily from r meson exchange and which respects the P symmetry, and the "weak" part, which comes from the same interaction responsible for B decay.
${}^{60}_{27}\text{Co}$

5.272 a

10.467 m
0.05859 MeV γ
2+
5+

0.31 MeV β 99.88%

1.48 MeV β 0.12%

1.1732 MeV γ

2.505 4+

2.158 2+

1.332 2+

1.3325 MeV γ 0+
Lee Yang Wu

Nobel prize in physics (1957)
CP violation

Makoto Kobayashi  Toshihide Maskawa

Nobel prize in physics (2008)
CPT theorem

- Preservation of CPT symmetry by all physical phenomena
- Any Lorentz invariant local quantum field theory with a Hermitian Hamiltonian must have CPT symmetry