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# Fundamentals in Nuclear Physics
# 原子核基礎

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# Nuclear decays and fundamental interactions (1)

# Four fundamental interactions

| interaction 相互作用 | exchanged particle (gauge boson) | decay 壊変 |
|---|---|---|
| gravity 重力 | graviton 重力子 | |
| weak 弱い相互作用 | $W^{\pm}, Z^0$ | beta decay |
| electromagnetic 電磁相互作用 | photon 光子 | gamma decay |
| strong 強い相互作用 | gluon グルーオン | |
| nuclear force 核力 | pion and other hadrons | |

alpha decay ← tunnel effect

壊変（崩壊）速度　自然幅

# Decay rate, natural width

probability to decay in an interval *dt*

$$dP = \frac{dt}{\tau} = \lambda dt$$

decay rate 壊変（崩壊）速度

mean life time 平均寿命

number of unstable nuclei $N(t) = N(t=0)e^{-t/\tau}$

half life 半減期 $t_{1/2} = (\ln 2)\tau = 0.693\tau$

$$^7\mathrm{Li}\,(7.459\,\mathrm{MeV}) \ \rightarrow \ \mathrm{n}\,^6\mathrm{Li}, \ \ ^3\mathrm{H}\,^4\mathrm{He} \qquad \tau \ = \ 6\times 10^{-21} \ \mathrm{sec}$$

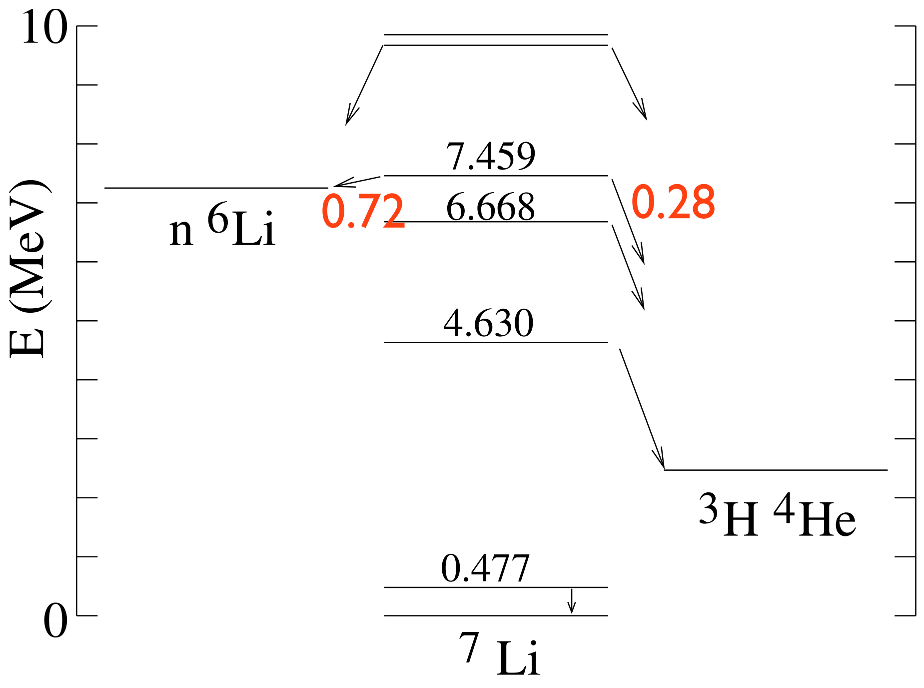
$^{76}\mathrm{Ge} \rightarrow {}^{76}\mathrm{Se}\,2\mathrm{e}^-\,2\bar{\nu}_e \qquad t_{1/2} = 1.78\times 10^{21} \ \mathrm{yr}$ > $10^{11}$ × (age of universe) !

An unstable particle has an energy uncertainty or “natural width”

$$\Gamma = \hbar\lambda = \frac{\hbar}{\tau} = \frac{6.58\times 10^{-22}\,\mathrm{MeV\ sec}}{\tau}$$

# 分岐比
# Branching ratio

- Often, an unstable state (nucleus, isotope) has more than one decay channels.



channel *k*

branching ratio

$$B_k$$

$$\sum_k B_k = 1$$

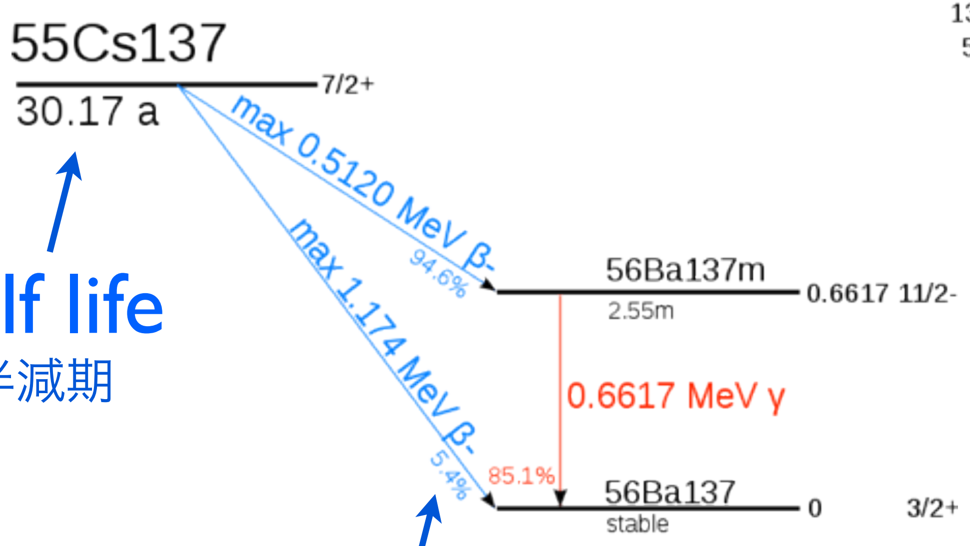
partial decay rate

$$\lambda_k = B_k \lambda \qquad \sum_k \lambda_k = \lambda$$

partial width 部分幅

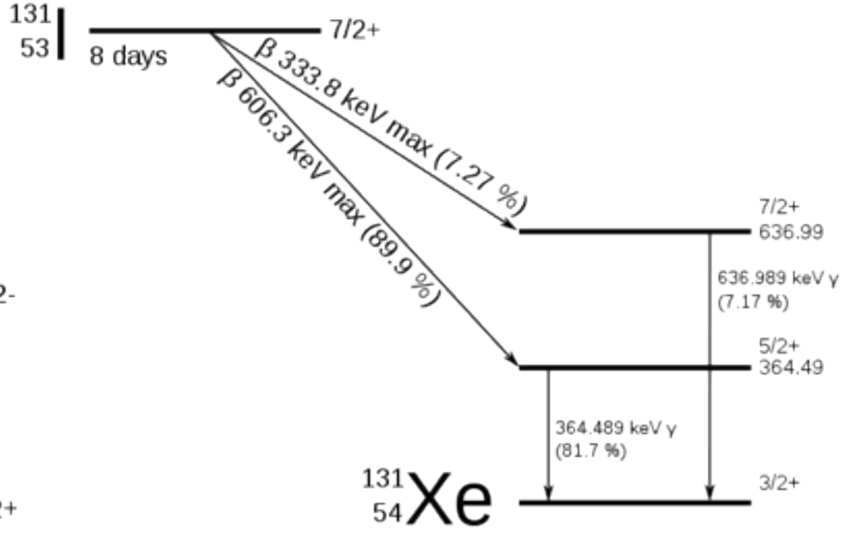
$$\Gamma_k = B_k \Gamma \qquad \sum_k \Gamma_k = \Gamma$$

# 壊変図
# Decay diagram



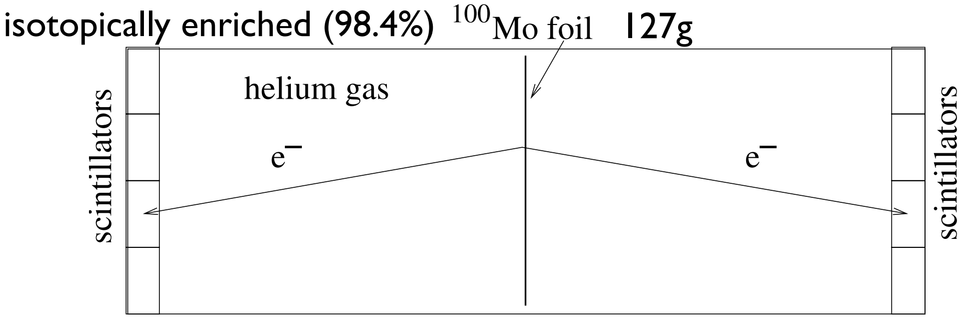
half life
半減期

branching ratio
分岐比

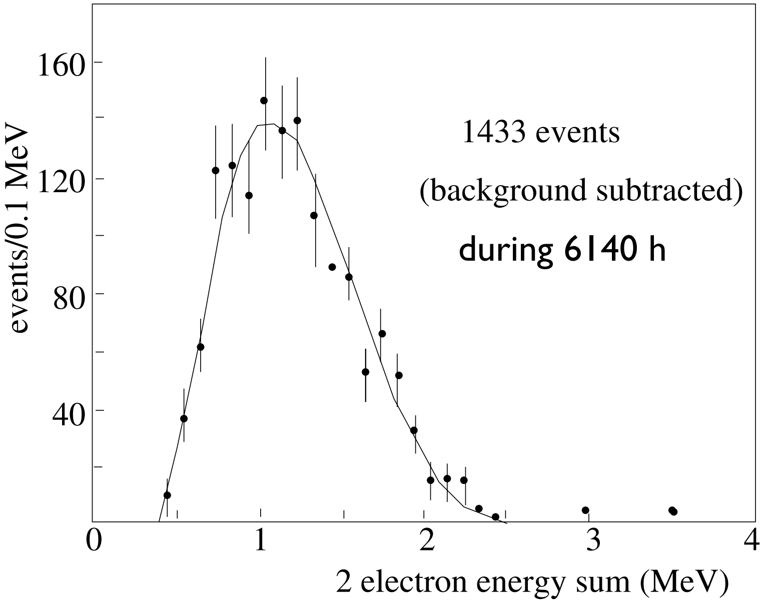


# Measurement of half life 半減期の測定

## τ > $10^8$ yr (α decay, double β decay)



isotopically enriched (98.4%) $^{100}$Mo foil 127g



during 6140 h

- still present on Earth
- can be chemically and isotopically isolated in macroscopic quantity
- detected decays, quantity → lifetime

$$^{100}\mathrm{Mo} \longrightarrow {}^{100}\mathrm{Ru}\, 2e^-\ 2\bar{\nu}_e$$ double β decay

half-life: (0.95±0.11)×$10^{19}$ yr

## 10 min $< \tau <$ $10^8$ yr (α decay, β decay)

- no longer present on Earth and must be produced in nuclear reactions
- purify chemically or isotopically
- detect decays and derive $\tau$

## $10^{-10}$ s $< \tau <$ $10^3$ s (α decay, β decay, γ decay)

- chemical and isotopic purification impossible
- particles produced in nuclear reactions, slowed down, and stopped
- detect decays and derive $\tau$

## $\tau <$ $10^{-10}$ s (γ decay, dissociation)

- standard timing techniques not applicable
- a variety of ingenious techniques: Doppler-shift attenuation method, Mössbauer spectroscopy

壊変速度の計算式

# Formula for decay rates

interaction 変化を引き起こす相互作用

**decay** $a \xrightarrow{T} \underbrace{b_1 + b_2 + \cdots + b_N}$

静止質量 rest mass $M$

energy $E = Mc^2$

$|f\rangle$ state of final particles 終状態

**decay rate** probability per unit time that $a$ decays into $f$

粒子 $a$ が単位時間に状態 $f$ に壊変する確率

$$\lambda_{a\to f} = \frac{2\pi}{\hbar} \underbrace{|\langle f|T|a\rangle|^2}_{\text{transition matrix element}} \, \delta \underbrace{\left( Mc^2 - \sum_j E_j \right)}_{\text{energy conservation}}$$

transition matrix element 遷移行列要素

energy conservation エネルギー保存

Fermi's golden rule

フェルミの黄金則

# Gamma decay

ガンマ壊変（崩壊）

# Energetics

エネルギーについての考察

**gamma decay**

$$\mathrm{A}^* \rightarrow \mathrm{A} + \gamma$$

gamma ray　ガンマ線

spontaneous emission　自然放出

unstable high-energy state

(stable) low-energy state

$$m_{A*} > m_A \qquad m_{A*} - m_A \ll m_A$$

momentum conservation $p = \dfrac{E_\gamma}{c}$ 運動量保存

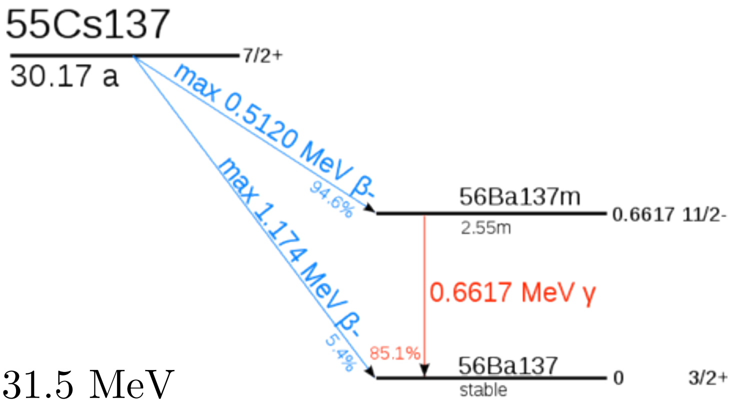
energy conservation　エネルギー保存

$$E_\gamma + \frac{p^2}{2m_A} = (m_{A*} - m_A)\,c^2$$

recoil energy (energy loss)

反跳エネルギー（エネルギー損失）

$$E_R = \frac{E_\gamma^2}{2m_A c^2} \qquad m_A c^2 \simeq A \times 931.5\ \mathrm{MeV}$$



$E_R \ll E_\gamma$ $\quad E_\gamma \simeq (m_{A*} - m_A)\,c^2$ $\quad$ but $E_R > \Gamma$ in general

Emitted gamma rays are not resonantly re-absorbed by other nuclei in gases

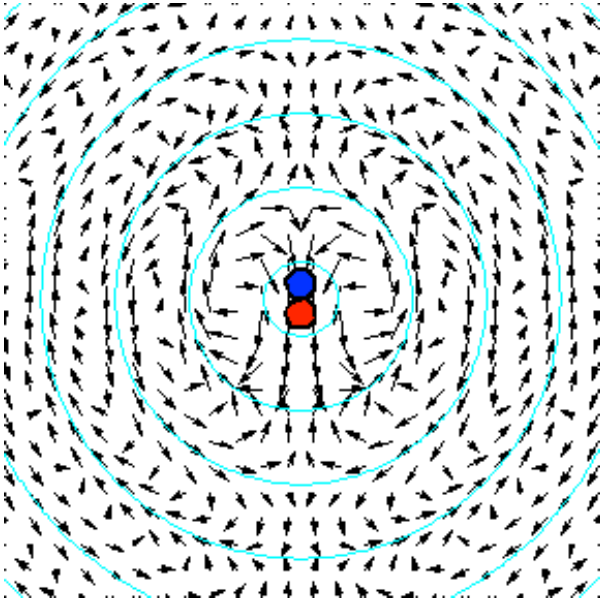
### 電気双極子遷移

# Electric-dipole transitions

Classical image　古典電磁気学的なイメージ

radiation from an oscillating electric dipole

振動する電気双極子からの古典的な放射



http://www.eto.titech.ac.jp/contents/sub04/chapter02.html

Quantum mechanically　量子力学的には

rate $\lambda_{i\to f} = \frac{4\alpha}{3}\frac{q^2}{e^2}\frac{E_\gamma^3}{\hbar^3 c^2}|\langle f|\mathbf{r}|i\rangle|^2$

fine-structure constant
微細構造定数 $\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \simeq \frac{1}{137}$

$$\langle f|\mathbf{r}|i\rangle = \int d^3\mathbf{r}\,\psi_f^*(\mathbf{r})\mathbf{r}\psi_i(\mathbf{r})$$

**Atomic transition**

$$\hbar\omega \sim \mathrm{eV} \quad \langle r\rangle \sim 10^{-10}\,\mathrm{m} \quad \tau \sim 10^{-9} - 10^{-7}\,\mathrm{s} \quad \Gamma = \hbar/\tau \sim 10^{-7}\,\mathrm{eV} \ll \hbar\omega$$
$$\gg E_R = E_\gamma^2/(2m_A c^2) \sim 10^{-9}\,\mathrm{eV}$$

**Nuclear transition** $\langle r\rangle \sim A^{1/3}10^{-15}\,\mathrm{m}$ ➡ $\lambda(E1) \sim \frac{\alpha E_\gamma^3}{\hbar}\left(\frac{A^{1/3}\,\mathrm{fm}}{\hbar c}\right)^2$

$$E_\gamma \sim \mathrm{MeV} \quad \tau \sim 10^{-17} - 10^{-15}\,\mathrm{s} \quad \Gamma \sim 10\,\mathrm{eV} \ll E_\gamma$$

多重極遷移

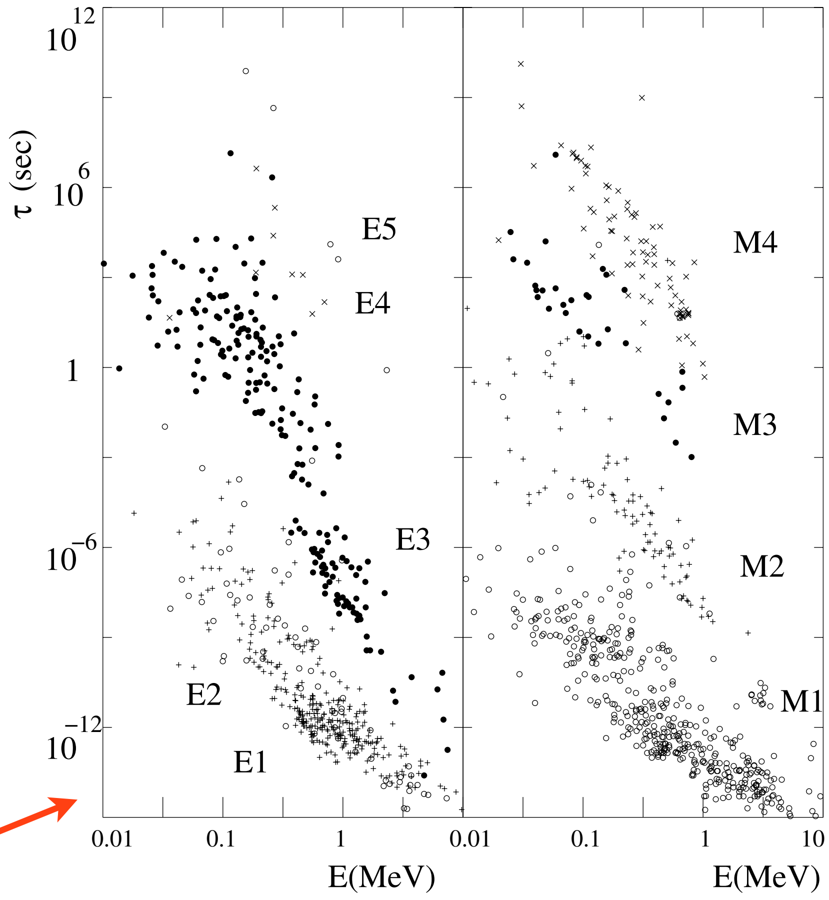
# Higher multi-pole transitions

Often, electric-dipole (E1) decay is forbidden. $\langle f|\mathbf{r}|i\rangle = 0$

➡ may still decay radiatively by higher-order and slower processes

**Table 4.1.** Selection rules for radiative transitions

| type | symbol | angular momentum change $\|\Delta J\| \leq$ | parity change |
|---|---|---|---|
| electric dipole | E1 | 1 | yes |
| magnetic dipole | M1 | 1 | no |
| electric quadrupole | E2 | 2 | no |
| magnetic quadrupole | M2 | 2 | yes |
| electric octopole | E3 | 3 | yes |
| magnetic octopole | M3 | 3 | no |
| electric 16-pole | E4 | 4 | no |
| magnetic 16-pole | M4 | 4 | yes |



Lifetime of excited nuclear states as a function of $E_\gamma$ for various multipoles

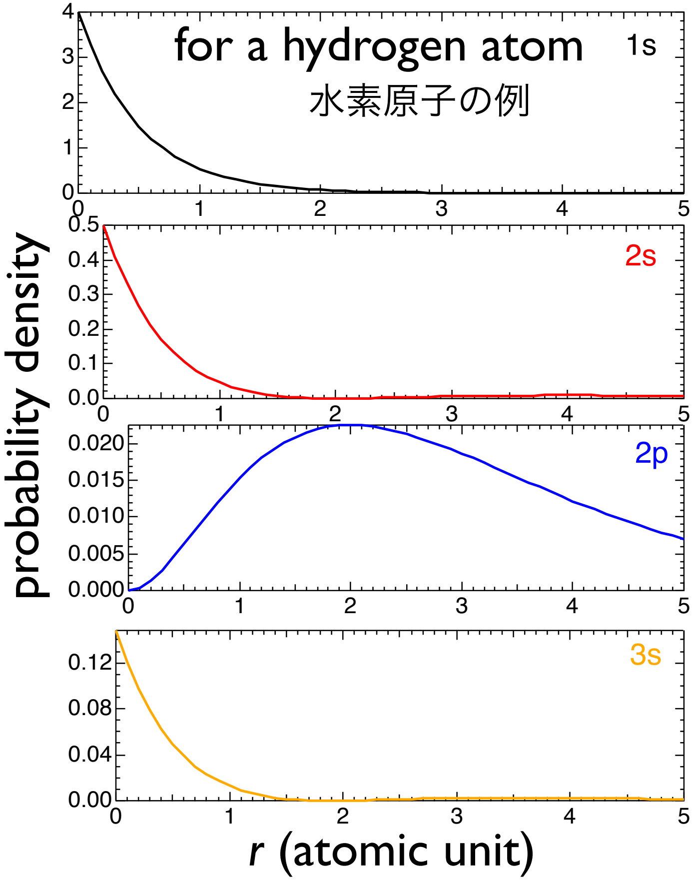
内部転換

# Internal conversion

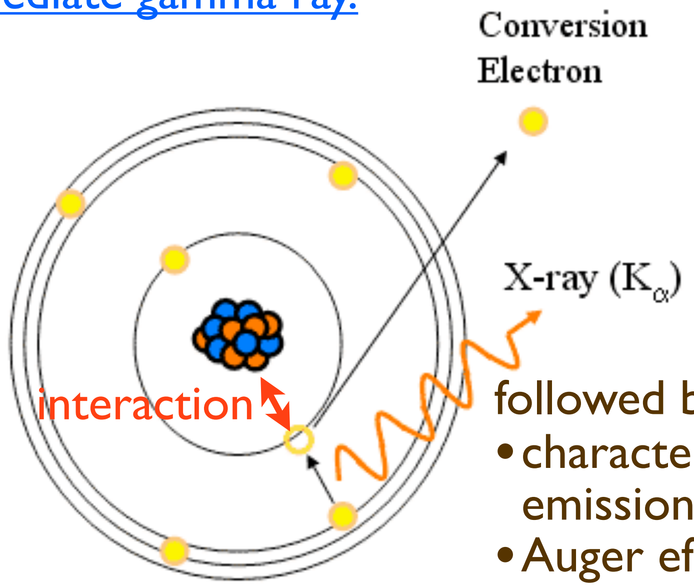
An excited nucleus can interact with an electron in one of the lower atomic orbitals, causing the electron to be emitted (ejected) from the atom.

s-electrons have finite probability density at the nuclear position.

s軌道の電子は、原子核の位置で存在確率が有限



The electron may couple to the excited state of the nucleus and take the energy of the nuclear transition <u>directly, without an intermediate gamma ray.</u>



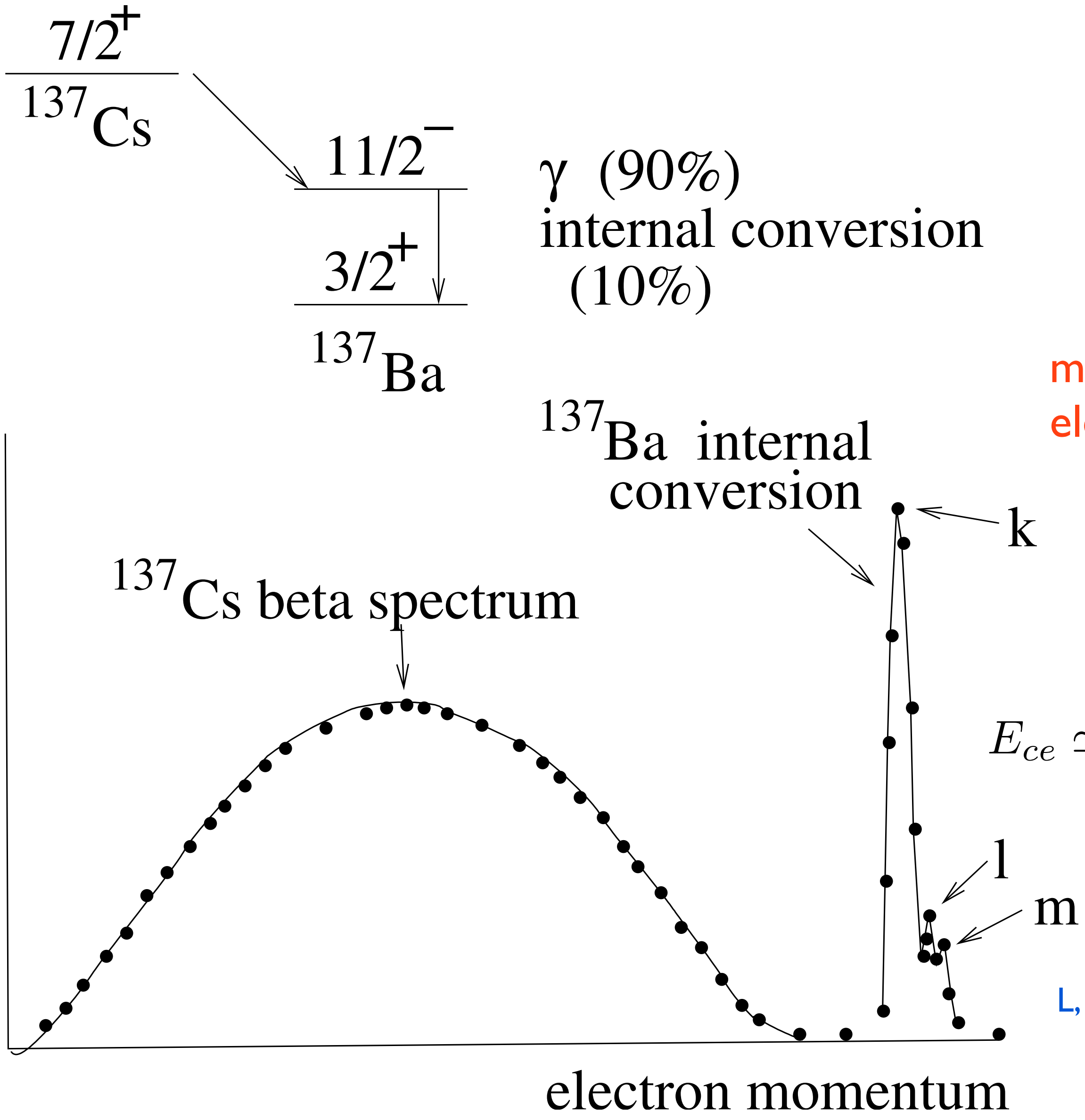
followed by

- characteristic x-ray emission 特性Ｘ線放出
- Auger effect オージェ効果

Energy of the conversion electron

$$E_{ce} \simeq (m_{A*} - m_A)\,c^2 - E_b \simeq E_\gamma - E_b$$

binding energy of the electron



most internal conversion electrons from the *K* shell

K殻からの内部転換電子が支配的

$$E_{ce} \simeq (m_{A*} - m_A)\,c^2 - E_b$$

ejection from higher orbitals generally less probable

L, M殻からの放出は一般的に少ない

メスバウアー効果

# Mössbauer effect

recoil energy (energy loss)

反跳エネルギー（エネルギー損失）

$$E_R = \frac{E_\gamma^2}{2m_A c^2}$$

Emitted gamma rays are not resonantly re-absorbed by other nuclei in gases.

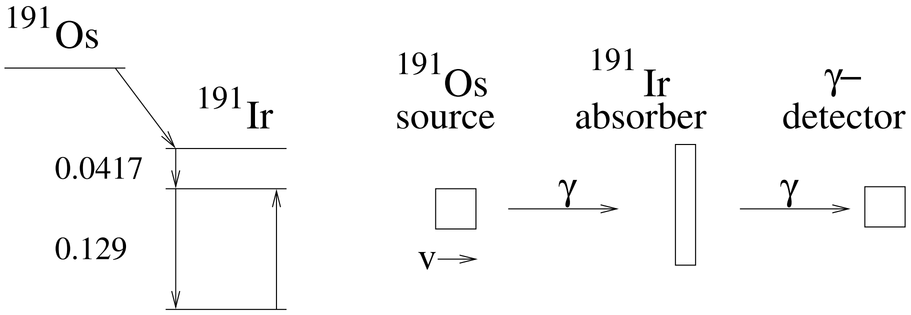
↓ but ...

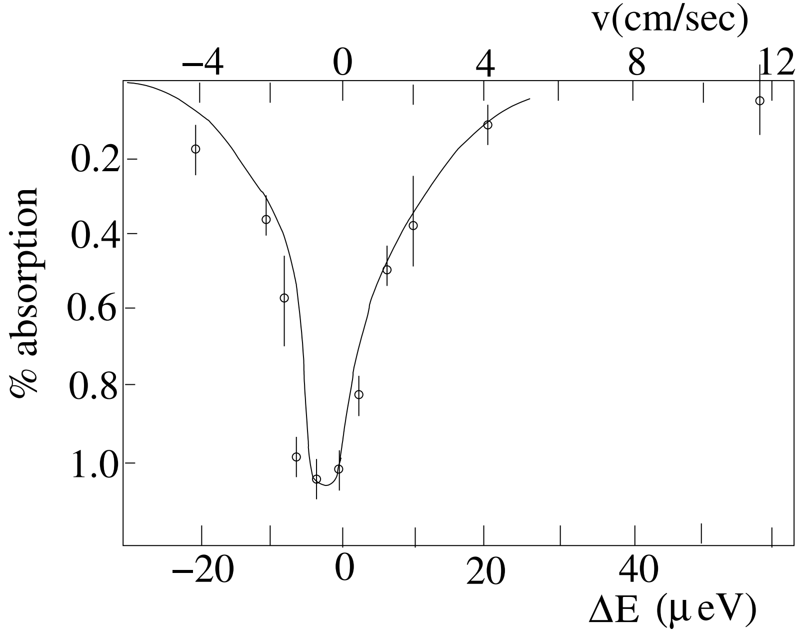
Inverse transition (resonant re-absorption) possible when

- nuclear recoil is suppressed in a crystal (“very very large $m_A$”) ← Mössbauer effect (discovered in 1957)
- the excited nucleus decays in flight with the Doppler effect compensating the nuclear recoil

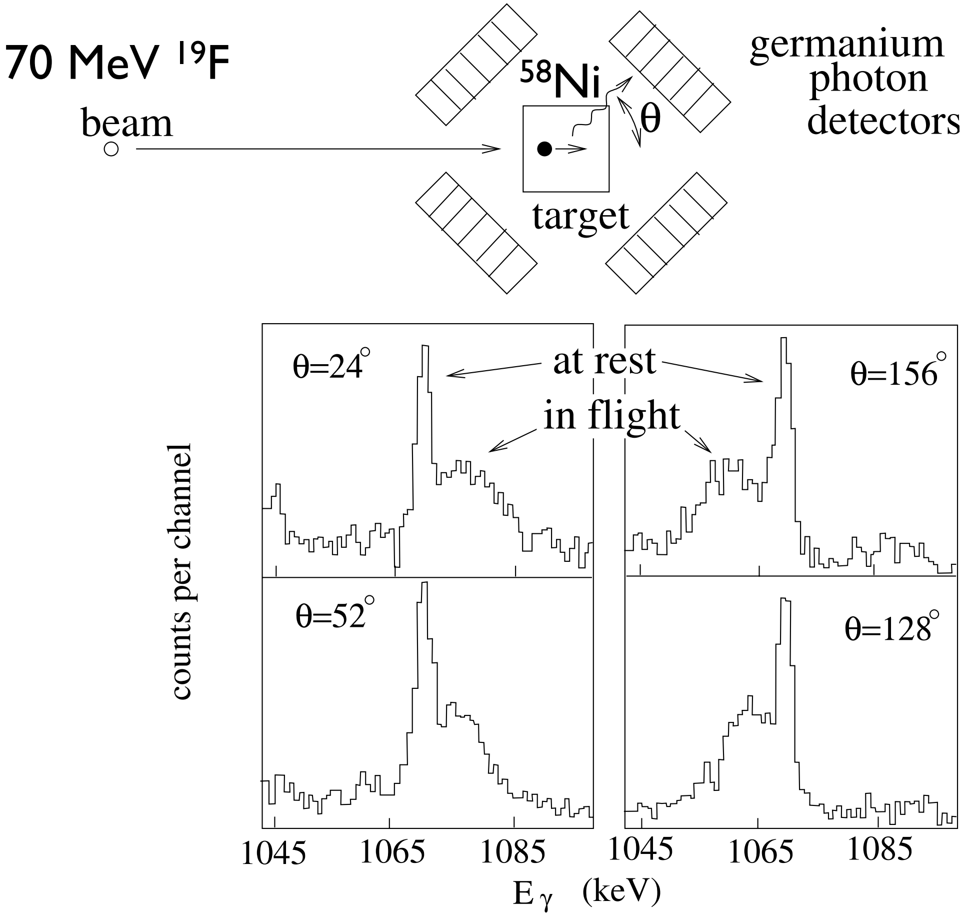
# メスバウアー分光による寿命測定

# Mössbauer spectroscopy





# Doppler-shift attenuation method



$^{74}$Br 1068 keV gamma-ray → 0.25 ps lifetime

メスバウアー効果　ドップラーシフト

*Mössbauer effect + Doppler shift* →

一般相対性理論の検証

# Test of Albert Einstein's theory of general relativity

by Pound and Rebka, 1959

- Gravitational red shift of light
- Clocks run differently at different places in a gravitational field

## *Gravitational shift*

$$h(f_r - f_e) = mgH$$

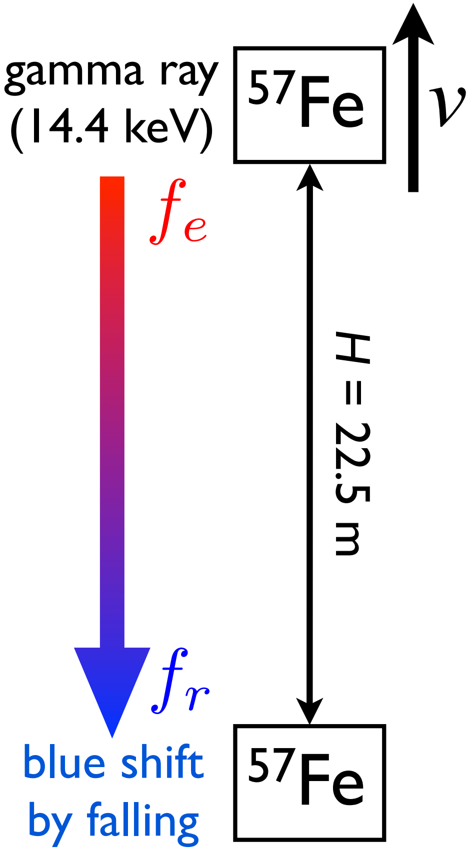
$$hf_e = mc^2$$

$$\frac{f_r}{f_e} = 1 + \frac{gH}{c^2}$$

## *Doppler shift*

$$\frac{f_r}{f_e} = \sqrt{\frac{1 - v/c}{1 + v/c}} \approx 1 - \frac{v}{c}$$

→ $$v = \frac{gH}{c} = 7.36 \times 10^{-7}\,\mathrm{m/s}$$



Jefferson laboratory
(Harvard University)