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Dielectric response in Bloch's hydrodynamic model of an electron–ion plasma

K. Ishikawa, B.U. Felderhof*

Institut für Theoretische Physik A, R.W.T.H. Aachen, Templergraben 55, 52056 Aachen, Germany

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Abstract

The linear response of an electron–ion plasma to an applied oscillating electric field is studied within the framework of Bloch's classical hydrodynamic model. The ions are assumed to be fixed in space and distributed according to a known probability distribution. The linearized equations of motion for electron density and flow velocity are studied with the aid of a multiple scattering analysis and cluster expansion. This allows systematic reduction of the many-ion problem to a composition of few-ion problems, and shows how the longitudinal dielectric response function can in principle be calculated. © 1998 Elsevier Science B.V. All rights reserved.

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1. Introduction

The theory of dielectric response of a plasma must deal simultaneously with two distinct features, the interaction between fast electrons roaming through the whole volume, and the strong perturbation of the electron motion caused by the slow ions. For simplicity we immobilize the ions altogether and consider them to be fixed in space with known random distribution. The Coulomb interaction between electrons has a strong effect, causing collective electron motion. In the following we use Bloch's classical hydrodynamic equations [1] to describe the motion of the electron gas for each fixed configuration of ions. An average over the distribution of ion configurations provides the mean electronic response to an applied oscillating electric field.

In earlier work a general formalism was developed to deal with the linear dielectric response of an electron–ion plasma [2]. It turns out that for Bloch's hydrodynamic

* Corresponding author. Tel.: +49-241-807019; fax: +49-241-8888188; e-mail: ufelder@physik.rwth-aachen.de.

model it is desirable to develop a separate approach. The linearized hydrodynamic equations can be cast in a form which leads naturally to a multiple scattering analysis. Subsequently, the average response can be studied by the method of cluster expansion. The resulting formalism is more transparent than that developed for the general class of models [2].

For simplicity we consider only electrostatics, and the corresponding longitudinal dielectric response function. Bloch's hydrodynamic model can be combined with the full set of Maxwell's equations to include the radiation field. A more elaborate analysis would yield also the transverse dielectric response function.

The equation of state relating local electron pressure and density is not specified. If the equation of state is identified with that of an ideal Fermi-gas, then the equilibrium situation corresponds to the Thomas–Fermi model. The zero temperature version of that model was used by Ball et al. [3] to study photoabsorption by an atom. The calculation can be extended to finite temperature and to an ion immersed in a plasma. The theory presented below provides the method to deal with the dielectric response of the entire many-ion plasma.

Bloch's hydrodynamic model has obvious shortcomings. The dynamics is classical and only collective motion is treated. Nonetheless, the model is important, since it leads to feasible calculations. A study of the model can serve as a benchmark and as a guide to more detailed analysis of the electron-ion plasma.

2. Plasma model

We study the linear response of an electron–ion plasma to an applied oscillating electric field within the framework of Bloch's hydrodynamic model [1]. The ions are assumed to be identical and fixed in space with centers at positions $\mathbf{R}_1, \dots, \mathbf{R}_N$. Each ion has a radially symmetric charge distribution consisting of a nuclear point charge at the center, and possibly an inert cloud of tightly bound core electrons. Thus the total ion charge density is

$$\rho_f(\mathbf{r}; \mathbf{X}) = e \sum_{j=1}^N F(|\mathbf{r} - \mathbf{R}_j|), \quad (2.1)$$

where $\mathbf{X} = (\mathbf{R}_1, \dots, \mathbf{R}_N)$ denotes the ion configuration, and $eF(|\mathbf{r} - \mathbf{R}_j|)$ is the charge density of ion j . The ions are neutralized by a gas of mobile electrons. The entire plasma is confined to a volume Ω of simple shape. Some of the mobile electrons are bound to the ions, others move freely throughout the volume Ω . At an appropriate stage of the calculation we take the thermodynamic limit $N \rightarrow \infty, \Omega \rightarrow \infty$ at constant ion number density $n_i = N/\Omega$.

The gas of mobile electrons is described hydrodynamically by its number density $n(\mathbf{r}, t)$ and flow velocity $\mathbf{v}(\mathbf{r}, t)$. These quantities satisfy the equations of motion

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{v}) = 0, \quad nm \frac{d\mathbf{v}}{dt} = -\nabla p + ne\nabla\phi, \quad (2.2)$$

where m is the electron mass, $-e$ is the electron charge, and d/dt is the substantial derivative $\partial/\partial t + (\mathbf{v} \cdot \nabla)$. The pressure p is assumed to be related to the number density n by a known equation of state $p = p(n)$. Furthermore, $\phi(\mathbf{r}, t)$ is the electrostatic potential, governed by Poisson's equation

$$\nabla^2 \phi = 4\pi n e - 4\pi \rho_f - 4\pi \rho_{\text{ex}}, \quad (2.3)$$

where $\rho_{\text{ex}}(\mathbf{r}, t)$ is the external charge density corresponding to the applied electric field.

We consider first the static equilibrium situation in the absence of the external charge density ρ_{ex} . In this situation the flow velocity \mathbf{v} vanishes, and the electron number density $n_0(\mathbf{r}; \mathbf{X})$ is time-independent and depends parametrically on the ion configuration \mathbf{X} . The corresponding pressure $p_0(\mathbf{r}; \mathbf{X})$ and electrostatic potential $\phi_0(\mathbf{r}; \mathbf{X})$ also depend on configuration and satisfy

$$\nabla p_0 - n_0 e \nabla \phi_0 = 0. \quad (2.4)$$

This equation is solved by

$$\mu_0 - e\phi_0 = \text{const.}, \quad (2.5)$$

where $\mu_0(\mathbf{r}; \mathbf{X})$ is the chemical potential, related to the pressure by the equation of state. The constant on the right hand side in Eq. (2.5) is adjusted in such a way that charge neutrality is satisfied. Charge neutrality implies

$$\int_{\Omega} n_0(\mathbf{r}; \mathbf{X}) d\mathbf{r} = NZ_f, \quad (2.6)$$

where $Z_f e$ is the charge of a single ion,

$$Z_f = \int F(\mathbf{r}) d\mathbf{r}. \quad (2.7)$$

If the equation of state is identified with that of an ideal Fermi-gas, then the equilibrium situation corresponds to the Thomas–Fermi model.

Next we consider the linear response to the electric field $\mathbf{E}_{\text{ex}}(\mathbf{r}, t) = -\nabla \phi_{\text{ex}}$, generated by the external charge density ρ_{ex} . The linearized equations of motion for the density deviation $n_1(\mathbf{r}, t)$, defined by

$$n = n_0 + n_1, \quad (2.8)$$

and the flow velocity $\mathbf{v}_1(\mathbf{r}, t)$ then read

$$\begin{aligned} \frac{\partial n_1}{\partial t} + \nabla \cdot (n_0 \mathbf{v}_1) &= 0, & m \frac{\partial \mathbf{v}_1}{\partial t} &= -\nabla \mu_1 + e \nabla \phi_1, \\ \nabla^2 \phi_1 &= 4\pi e n_1 - 4\pi \rho_{\text{ex}}, \end{aligned} \quad (2.9)$$

with first order chemical potential

$$\mu_1 = \vartheta_0 n_1, \quad \vartheta_0 = \left. \frac{\partial \mu}{\partial n} \right|_{n_0} = \frac{1}{n_0} \left. \frac{\partial p}{\partial n} \right|_{n_0}. \quad (2.10)$$

We may solve by putting

$$v_1 = \nabla S_1, \tag{2.11}$$

where $S_1(\mathbf{r}, t)$ is the first order streaming potential, which satisfies

$$m \frac{\partial S_1}{\partial t} = -\vartheta_0 n_1 + e\phi_1. \tag{2.12}$$

Since the equations are linear, we can consider an external charge density $\rho_{\text{ex}}(\mathbf{r}, t) = \rho_{\text{ex}\omega}(\mathbf{r}) \exp(-i\omega t)$ which varies harmonically in time. We put

$$\begin{aligned} n_1(\mathbf{r}, t) &= n_\omega(\mathbf{r}) \exp(-i\omega t), & \phi_1(\mathbf{r}, t) &= \phi_\omega(\mathbf{r}) \exp(-i\omega t), \\ S_1(\mathbf{r}, t) &= S_\omega(\mathbf{r}) \exp(-i\omega t). \end{aligned} \tag{2.13}$$

The equations for the Fourier amplitudes then read

$$\begin{aligned} -i\omega n_\omega + \nabla \cdot (n_0 \nabla S_\omega) &= 0, \\ -i\omega m S_\omega + \vartheta_0 n_\omega - e\phi_\omega &= 0, & \nabla^2 \phi_\omega &= 4\pi e n_\omega - 4\pi \rho_{\text{ex}\omega}. \end{aligned} \tag{2.14}$$

The solution of Poisson’s equation may be expressed as

$$\phi_\omega(\mathbf{r}) = \phi_{\text{ex}\omega}(\mathbf{r}) - e \int \frac{n_\omega(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d\mathbf{r}'. \tag{2.15}$$

Here we assume vacuum outside the plasma. The first two equations in Eq. (2.14) may therefore be rewritten as

$$\begin{aligned} -i\omega n_\omega + \nabla \cdot (n_0 \nabla S_\omega) &= 0, \\ -i\omega S_\omega + \frac{1}{m} \vartheta_0 n_\omega + \frac{e^2}{m} \int \frac{n_\omega(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d\mathbf{r}' &= \frac{e}{m} \phi_{\text{ex}\omega}. \end{aligned} \tag{2.16}$$

If we introduce the two-component fields

$$\Psi_\omega = \begin{pmatrix} n_\omega \\ S_\omega \end{pmatrix}, \quad U_\omega = \begin{pmatrix} 0 \\ \frac{e}{m} \phi_{\text{ex}\omega} \end{pmatrix}. \tag{2.17}$$

we can cast Eq. (2.16) in the form

$$(-i\omega + \mathcal{L}) \Psi_\omega = U_\omega \tag{2.18}$$

with a linear operator \mathcal{L} . The operator depends on ion configuration \mathbf{X} via the equilibrium profiles $n_0(\mathbf{r}; \mathbf{X})$ and $\vartheta_0(\mathbf{r}; \mathbf{X})$, but the right-hand side U_ω is independent of configuration. Hence Eq. (2.18) has a familiar structure. We can apply statistical methods to find the average behavior of the solution.

The equilibrium profiles n_0 and ϑ_0 may be expressed as

$$n_0(\mathbf{r}; \mathbf{X}) = n_0^{(0)} + \delta n_0(\mathbf{r}; \mathbf{X}), \quad \vartheta_0(\mathbf{r}; \mathbf{X}) = \vartheta_0^{(0)} + \delta \vartheta_0(\mathbf{r}; \mathbf{X}), \tag{2.19}$$

with mean electron density $n_0^{(0)}$ given by

$$n_0^{(0)} = n_i Z_f, \tag{2.20}$$

so that, from Eq. (2.6),

$$\int_{\Omega} \delta n_0(\mathbf{r}; \mathbf{X}) d\mathbf{r} = 0. \quad (2.21)$$

The constant $\vartheta_0^{(0)}$ is related to $n_0^{(0)}$ by the equation of state according to Eq. (2.10). Corresponding to the separation of terms in Eq. (2.19) we can write the linear operator \mathcal{L} as the sum

$$\mathcal{L}(\mathbf{X}) = \mathcal{L}_0 + \mathcal{V}(\mathbf{X}) \quad (2.22)$$

with \mathcal{L}_0 corresponding to the values $n_0^{(0)}$ and $\vartheta_0^{(0)}$, and with the linear operator $\mathcal{V}(\mathbf{X})$ depending on configuration. We define the corresponding polarization field

$$\mathbf{\Pi}_\omega = \mathcal{V}(\mathbf{X})\Psi_\omega. \quad (2.23)$$

The probability distribution of configurations $W(\mathbf{X})$ is assumed known. Averaging Eq. (2.18) over this distribution we find

$$(-i\omega + \mathcal{L}_0) \langle \Psi_\omega \rangle + \langle \mathbf{\Pi}_\omega \rangle = \mathbf{U}_\omega. \quad (2.24)$$

This becomes a closed equation for the average field $\langle \Psi_\omega \rangle$ if the average polarization $\langle \mathbf{\Pi}_\omega \rangle$ can be expressed in terms of $\langle \Psi_\omega \rangle$ by means of a constitutive equation

$$\langle \mathbf{\Pi}_\omega(\mathbf{r}) \rangle = \int \Xi(\mathbf{r}, \mathbf{r}', \omega) \langle \Psi_\omega(\mathbf{r}') \rangle d\mathbf{r}'. \quad (2.25)$$

The kernel $\Xi(\mathbf{r}, \mathbf{r}', \omega)$ is expected to be of short range, and to become translationally invariant in the thermodynamic limit. The calculation of the average linear response of the plasma hinges on the determination of the constitutive kernel Ξ .

3. Response functions

The separation of the linear operator \mathcal{L} in Eq. (2.22) into two terms suggests that the first term \mathcal{L}_0 can be regarded as a first approximation to \mathcal{L} . This corresponds to a uniformization of the system, with the ions smeared out into a uniform background. The second term $\mathcal{V}(\mathbf{X})$ takes account of the discrete nature of the ions. We consider first the one-component plasma described by the operator \mathcal{L}_0 . In the thermodynamic limit this plasma fills all space with uniform electron density $n_0^{(0)}$ in the unperturbed state. We find the properties of the actual plasma by treating the second term $\mathcal{V}(\mathbf{X})$ as a perturbation.

The linear response of the plasma will be characterized by the wavenumber- and frequency-dependent longitudinal dielectric function $\varepsilon_L(q, \omega)$. This is defined from the relation between average potential and average charge density of the plasma perturbed by a plane wave external charge density

$$\rho_{\text{ex}}(\mathbf{r}, t) = \rho_{\text{ex}q\omega} \exp[\mathbf{i}\mathbf{q} \cdot \mathbf{r} - i\omega t]. \quad (3.1)$$

From the response in the thermodynamic limit one finds the dielectric function [4]

$$\epsilon_f(q, \omega) = 1 - \frac{4\pi \langle \rho_{q\omega} \rangle}{q^2 \langle \phi_{q\omega} \rangle}, \tag{3.2}$$

where $\rho_{q\omega}$ is the plane wave amplitude of the perturbed electron charge density $\rho_1(\mathbf{r}, t) = -en_1(\mathbf{r}, t)$. We denote the electron charge density for the uniform plasma by $\rho_\omega^{(0)}$, and the corresponding potential by $\phi_\omega^{(0)}$. These are related by

$$\nabla^2 \phi_\omega^{(0)} = -4\pi\rho_\omega^{(0)} - 4\pi\rho_{ex\omega}. \tag{3.3}$$

From Eq. (2.16) one finds for the corresponding dielectric function

$$\epsilon_f^{(0)}(q, \omega) = 1 - \frac{\omega_{p0}^2}{\omega^2 - q^2 s^2} \tag{3.4}$$

with coefficients

$$\omega_{p0}^2 = \frac{4\pi n_0^{(0)} e^2}{m}, \quad s^2 = \frac{n_0^{(0)} v_0^{(0)}}{m}. \tag{3.5}$$

Hence the susceptibility of the uniform plasma is

$$\chi_f^{(0)}(q, \omega) = \frac{-1}{4\pi} \frac{\omega_{p0}^2}{\omega^2 - q^2 s^2}. \tag{3.6}$$

The Fourier-transformed density–density response function $\chi_{nn}^{(0)}(q, \omega)$ is defined by [5]

$$\rho_{q\omega}^{(0)} = -e^2 \chi_{nn}^{(0)}(q, \omega) \phi_{exq\omega}. \tag{3.7}$$

From Eqs. (3.2) and (3.3) one finds

$$\frac{1}{\epsilon_f^{(0)}(q, \omega)} = 1 - \frac{4\pi e^2}{q^2} \chi_{nn}^{(0)}(q, \omega), \tag{3.8}$$

so that

$$\chi_{nn}^{(0)}(q, \omega) = -\frac{q^2}{4\pi e^2} \frac{\omega_{p0}^2}{\omega^2 - \omega_{p0}^2 - q^2 s^2}. \tag{3.9}$$

The relation between $\rho_\omega^{(0)}$ and the potential $\phi_\omega^{(0)}$ is

$$\rho_{q\omega}^{(0)} = -q^2 \chi_f^{(0)}(q, \omega) \phi_{q\omega}^{(0)}. \tag{3.10}$$

In the actual plasma the response functions are modified due to the discrete nature of the ions.

For given ion configuration \mathbf{X} the charge density and potential can be expressed as

$$\rho_\omega = \rho_\omega^{(0)} + \rho_\omega^{(1)}, \quad \phi_\omega = \phi_\omega^{(0)} + \phi_\omega^{(1)}. \tag{3.11}$$

Previously [2] an additional charge density $\delta\rho_\omega(\mathbf{X})$ was defined from the relation

$$\delta\rho_\omega(\mathbf{X}) = \rho_\omega(\mathbf{X}) - X^{(0)}(\omega) \phi_\omega(\mathbf{X}), \tag{3.12}$$

where $X^{(0)}(\omega)$ is the integral kernel corresponding to the factor $-q^2 \chi_r^{(0)}(q, \omega)$ in Eq. (3.10). The cluster expansion of the linear response was based on the linear relation between the additional charge density $\delta\rho_\omega(\mathbf{X})$ and the potential $\phi_\omega(\mathbf{X})$. Here we follow a different approach, and base the cluster expansion on the linear relation Eq. (2.23) between the two-component polarization Π_ω and the two-component field Ψ_ω . The advantage is that this relation is local, as can be seen from Eq. (2.16). It is obvious that the perturbation is localized on the ions. We need a single additional assumption, to be discussed in Section 4, which allows us to perform a multiple scattering expansion, an important tool in explicit calculations.

4. Superposition approximation

In this section we formulate a superposition approximation based on a consideration of the equilibrium profiles $n_0(\mathbf{r}; \mathbf{X})$ and $\vartheta_0(\mathbf{r}; \mathbf{X})$, and related profiles which can be found by partial averaging over the ion distribution function. The probability distribution $W(\mathbf{X})$ of ion configurations is assumed normalized to unity and symmetric in the labels $1, \dots, N$. The partial distribution function

$$n_i(1, \dots, s) = \frac{N!}{(N-s)!} \int \cdots \int d\mathbf{R}_{s+1} \cdots d\mathbf{R}_N W(1, \dots, N) \quad (4.1)$$

gives the probability of finding a configuration of s ions whatever the configuration of the remaining $N-s$ ions. Thus $n_i(1)$ is the density of ions, $n_i(1, 2)$ is the distribution of pairs of ions, etc. Correspondingly, the conditional partial average of the equilibrium electron density $n_0(\mathbf{r}; \mathbf{X})$ is defined by

$$\bar{n}_0(\mathbf{r}; 1, \dots, s) = \frac{N!}{(N-s)!} \int \cdots \int d\mathbf{R}_{s+1} \cdots d\mathbf{R}_N n_0(\mathbf{r}; \mathbf{X}) W(1, \dots, N) / n_i(1, \dots, s). \quad (4.2)$$

This is the mean equilibrium electron density at \mathbf{r} with the condition that s ions are centered at $\mathbf{R}_1, \dots, \mathbf{R}_s$. In particular, $\bar{n}_0(\mathbf{r}; 1)$ describes screening of an ion charge centered at \mathbf{R}_1 . We identify

$$\bar{n}_0(\mathbf{r}; 1, \dots, N) = n_0(\mathbf{r}; \mathbf{X}). \quad (4.3)$$

Similarly, we define

$$\bar{\vartheta}_0(\mathbf{r}; 1, \dots, s) = \frac{N!}{(N-s)!} \int \cdots \int d\mathbf{R}_{s+1} \cdots d\mathbf{R}_N \vartheta_0(\mathbf{r}; \mathbf{X}) W(1, \dots, N) / n_i(1, \dots, s), \quad (4.4)$$

and identify

$$\bar{\vartheta}_0(\mathbf{r}; 1, \dots, N) = \vartheta_0(\mathbf{r}; \mathbf{X}). \quad (4.5)$$

We can now consider a sequence of electron- s -ion systems [2] with $s=0, 1, \dots, N$ on the basis of Eq. (2.16). The electron-0-ion system is described by Eq. (2.16) with

$n_0(\mathbf{r}; \mathbf{X})$ and $\vartheta_0(\mathbf{r}; \mathbf{X})$ replaced by the uniform values $n_0^{(0)}$ and $\vartheta_0^{(0)}$, respectively. The modified equation describes the dynamics of a one-component plasma. The electron– s -ion system for $s > 0$ is described by Eq. (2.16) with $n_0(\mathbf{r}; \mathbf{X})$ replaced by $\bar{n}_0(\mathbf{r}; 1, \dots, s)$ and $\vartheta(\mathbf{r}; \mathbf{X})$ replaced by $\bar{\vartheta}_0(\mathbf{r}; 1, \dots, s)$. On account of Eqs. (4.3) and (4.5) the electron– N -ion system is identical with the original one. The definition of the electron– s -ion systems allows one to construct an approximate description of the original system with the aid of a cluster expansion.

We abbreviate the equations describing the linear response of the electron– s -ion system as

$$(-i\omega + \overline{\mathcal{L}}(1, \dots, s))\Psi_\omega(1, \dots, s) = U_\omega, \tag{4.6}$$

in analogy to Eq. (2.18). The linear operator $\overline{\mathcal{L}}(1, \dots, s)$ can be decomposed as in Eq. (2.22)

$$\overline{\mathcal{L}}(1, \dots, s) = \mathcal{L}_0 + \overline{\mathcal{V}}(1, \dots, s). \tag{4.7}$$

The additional assumption, alluded to at the end of Section 3, can now be formulated as the approximation

$$\overline{\mathcal{V}}(1, \dots, s) \approx \sum_{j=1}^s \overline{\mathcal{V}}(j), \tag{4.8}$$

with $\overline{\mathcal{V}}(j)$ calculated for the electron–1-ion system. We shall call this the superposition approximation. It may be expected to be valid, because the perturbation is due to a single ion where it is strong, and may be regarded as a sum of contributions from different ions where it is weak. With the superposition approximation Eq. (4.8) the dynamical equation Eq. (4.6) for the s -ion system may be solved formally by a multiple scattering expansion with the operator

$$G_{0\omega} = (-i\omega + \mathcal{L}_0)^{-1} \tag{4.9}$$

acting as Green function. In particular the single-ion scattering operator is defined by

$$\overline{T}(j) = \overline{\mathcal{V}}(j)[1 + G_{0\omega}\overline{\mathcal{V}}(j)]^{-1}. \tag{4.10}$$

The s -body scattering operator $\overline{T}(1, \dots, s)$, defined similarly with the perturbation $\overline{\mathcal{V}}(1, \dots, s)$, is approximated by the multiple scattering expansion defined as the sum of all scattering sequences for ions $1, \dots, s$, consisting of single-ion scattering operators $\overline{T}(j)$ connected by the Green function $G_{0\omega}$, with the condition that no two successive ion labels in a sequence are the same.

The superposition approximation allows one to decompose the linear response of the N -ion system into a sum of sequences of independent scattering events. In the calculation of the average linear response the method of cluster expansion may be applied. In the formulation of the cluster expansion [6,7], use is made of the multiple scattering expansion. Note that we allow spatial overlap in the range of the different operators $\overline{\mathcal{V}}(j)$ in Eq. (4.8). For the electron–ion plasma such overlap will in general be

present [8–10]. The decomposition into single-particle terms in Eq. (4.8) is sufficient for the multiple scattering expansion.

5. Cluster expansion

The cluster expansion will be described only briefly, since the formal structure described above allows one to apply the scheme as developed for general purposes [6,7]. In the expansion the constitutive kernel characterizing the average linear response of the system is decomposed into a sum of cluster integrals. The hope is that calculation of a subclass of relatively simple integrals suffices to get a good approximation to the actual response.

Since the response is symmetric in ion labels it suffices to restrict attention to scattering sequences in which the first ion has label 1. We indicate this selection by a factor $\chi(1)$ preceding the scattering operator. Thus the s -body scattering operator $\chi(1)\bar{T}(1, \dots, s)$ is the sum over the subclass of scattering sequences starting with label 1 on the left.

Rooted cluster operators $M(1; 2, \dots, s)$ may be defined successively as follows:

$$\begin{aligned}\chi(1)\bar{T}(1) &= M(\cdot), & \chi(1)\bar{T}(1, 2) &= M(1; 2) + M(1), \\ \chi(1)\bar{T}(1, 2, 3) &= M(1; 2, 3) + M(1; 2) + M(1; 3) + M(1), \dots, \text{ etc.}\end{aligned}\quad (5.1)$$

The relation between the field $\Psi_\omega(1, \dots, s)$ for the s -ion system and the field $\Psi_\omega^{(0)}$ for the 0-ion system is described by a linear operator

$$\Psi_\omega(1, \dots, s) = K(1, \dots, s)\Psi_\omega^{(0)}. \quad (5.2)$$

Cluster operators $L(1, \dots, s)$ are defined successively by

$$\begin{aligned}K(\emptyset) &= L(\emptyset) = I, & K(1) &= L(1) + L(\emptyset), \\ K(1, 2) &= L(1, 2) + L(1) + L(2) + L(\emptyset), \dots, \text{ etc.}\end{aligned}\quad (5.3)$$

Here \emptyset denotes the empty set, so $K(\emptyset)$ is the identity operator. The cluster expansion of the constitutive kernel $\Xi(\omega)$ is given by [6]

$$\Xi(\omega) = \sum_{s=1}^{\infty} \frac{1}{(s-1)!} \Xi_s(\omega) \quad (5.4)$$

with s -ion contribution

$$\Xi_s(\omega) = \int \cdots \int d\mathbf{R}_1 \cdots d\mathbf{R}_s \sum_{(B)} (-1)^{k-1} n_i(B_1) M(B_1) \cdots n_i(B_k) L(B_k), \quad (5.5)$$

where the sum is over all ordered partitions $(B) = (B_1 | B_2 | \cdots | B_k)$ of the labels $1, \dots, s$ into disjoint subsets with the condition that label 1 be in the first subset. Finally, the

thermodynamic limit $N \rightarrow \infty, \Omega \rightarrow \infty$ at constant $n_i = N/\Omega$ is taken. As a consequence, the cluster integrals become translationally invariant. The lowest order term reads

$$\Xi_1(\omega) = n_i \int d\mathbf{R}_1 M(1). \quad (5.6)$$

The operator is translationally invariant due to the integration over \mathbf{R}_1 . The two-ion term is given by

$$\Xi_2(\omega) = \int \int d\mathbf{R}_1 d\mathbf{R}_2 [n_i(1,2)M(1;2) - n_i(1)n_i(2)M(1)L(2)]. \quad (5.7)$$

The specification of the operator $M(1;2)$ requires the solution of Eq. (4.6) for $s=2$.

The probability distribution $W(\mathbf{X})$ may be assumed to incorporate a nonoverlap condition which implies that $W(\mathbf{X})$ vanishes for configurations \mathbf{X} for which any pair distance $|\mathbf{R}_j - \mathbf{R}_k|$ is less than a minimum distance $2a$, corresponding to a hard-core radius a . It has been shown [11] that a selection of terms from the cluster expansion corresponding to a virtual overlap region in each of the cluster integrals gives rise to the Clausius–Mossotti formula for a suspension of dielectric spheres. The overlap region $V_0(1|2|\dots|s)$ in the s -particle cluster integral corresponds to the restriction

$$|\mathbf{R}_j - \mathbf{R}_{j-1}| < 2a, \quad j = 2, 3, \dots, s. \quad (5.8)$$

The same selection of terms would provide an interesting approximation to the constitutive kernel of a dense plasma. The calculation would provide an approximation to the frequency-dependent longitudinal dielectric function $\epsilon_l(q, \omega)$, defined in Section 3.

6. Discussion

We have studied the linear response of an electron–ion plasma to an applied oscillating electric field within the framework of Bloch’s classical hydrodynamic model. To simplify the discussion we have considered only electrostatics, but the interaction with the radiation field can be included without difficulty. The theoretical model leads naturally to a treatment in which the discrete nature of the ions is regarded as perturbation of a related one-component plasma, in which the inert ions are smeared out into a uniform background. The discrete ions may be regarded as local modifications of the background, which act as scattering centers for the motion of the electron gas. In superposition approximation the perturbation is decomposed into a sum of single-ion contributions. The approximation permits a multiple scattering expansion of the many-ion response. A subsequent average over ion configurations can be performed with the aid of a cluster expansion. The resulting expressions for the response functions differ formally from those developed in a more general context [2]. The present formulation is more transparent and easier to apply.

For a dilute plasma the response can be calculated from a single-ion problem. The cluster expansion specifies the nature of this problem and shows in principle how

corrections to the low density result can be obtained. In earlier work [12] general properties of the photoabsorption cross section of a single ion in a plasma were studied within the framework of Bloch's hydrodynamic model. Explicit calculations were performed for a uniform sphere model of the ion [13]. A more realistic description, in which the ion is treated as a point charge and the electron gas is assumed to obey the Thomas–Fermi equation of state, requires elaborate analysis. We shall report separately on the explicit calculations for this case.

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